Estimation of NWP parameters using EnKF-based algorithms

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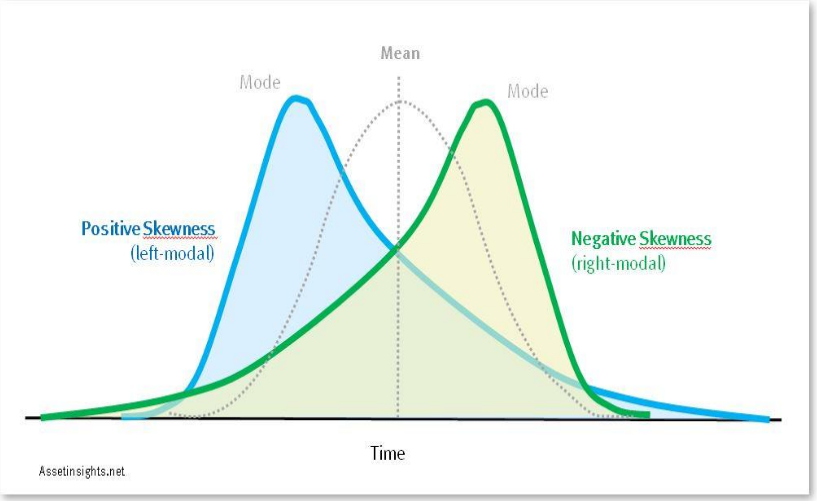
Motivation

Representation of clouds in convection permitting models is sensitive to NWP parameters that are often very crudely known.

Goal

Treat these parameters as uncertain and estimate them along with the state in order to:

- Reduce forecast errors
- Better capture the uncertainty of forecasts



Challenges

- Non-Gaussianity
- Violation of conservation laws

Research Question

What is more effective:

- Taking higher order moments into account (*Quadratic Filter, Hodyss 2012*)
- Satisfying conservation laws and physical bounds (*QP Ensemble, Janjic et al 2014*)

$$J(\delta x; v_i) := \delta x^T P^{-1} \delta x + [v_i - H\delta x]^T R^{-1} [v_i - H\delta x]$$
$$v_i = y + \varepsilon_i^o - H\left(\overline{x}^f + \varepsilon_i^f\right)$$

Algorithms

EnKF (Evensen, 1994)

$$x_i^a - x_i^f = \min_{\delta x} J(\delta x; v_i) = PH^T\left(HPH^T + R\right)^{-1} v_i$$

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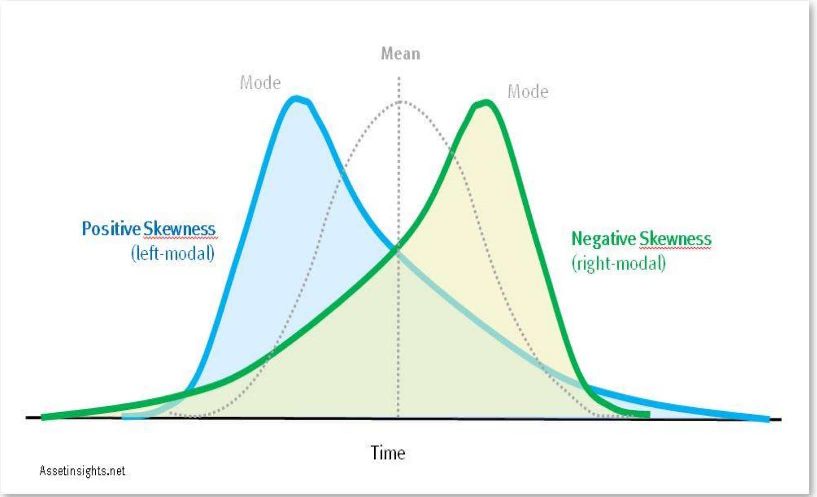

Algorithms

EnKF (Evensen, 1994)

$$x_i^a - x_i^f = \min_{\delta x} J(\delta x; v_i) = PH^T\left(HPH^T + R\right)^{-1} v_i$$

Quadratic Filter (Hodyss, 2012)

$$x_i^a - x_i^f = \tilde{P}H^T\left(H\tilde{P}H^T + \tilde{R}\right)^{-1} \tilde{v}_i$$

$$\tilde{P} = \begin{bmatrix} P & P_{skew} \\ P_{skew} & P_{kurt} \end{bmatrix} \quad \tilde{R} = \begin{bmatrix} R & R_{skew} \\ R_{skew} & R_{kurt} \end{bmatrix} \quad \tilde{v}_i = \begin{bmatrix} v_i \\ v_i \cdot v_i \end{bmatrix}$$

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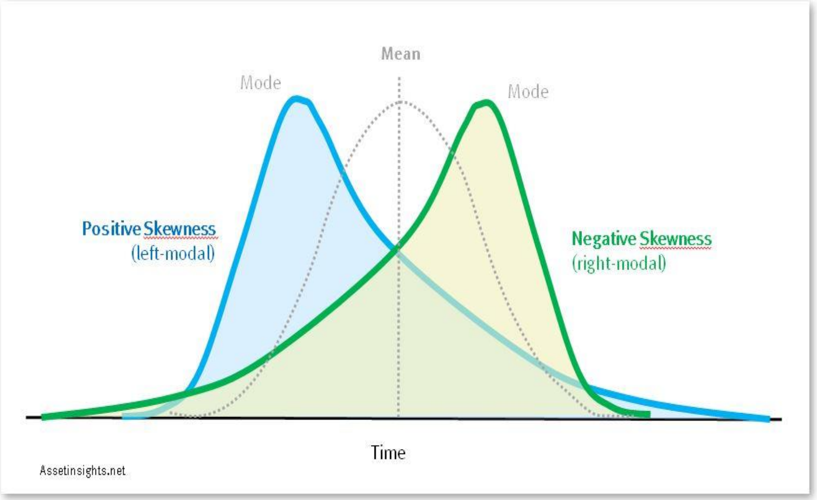


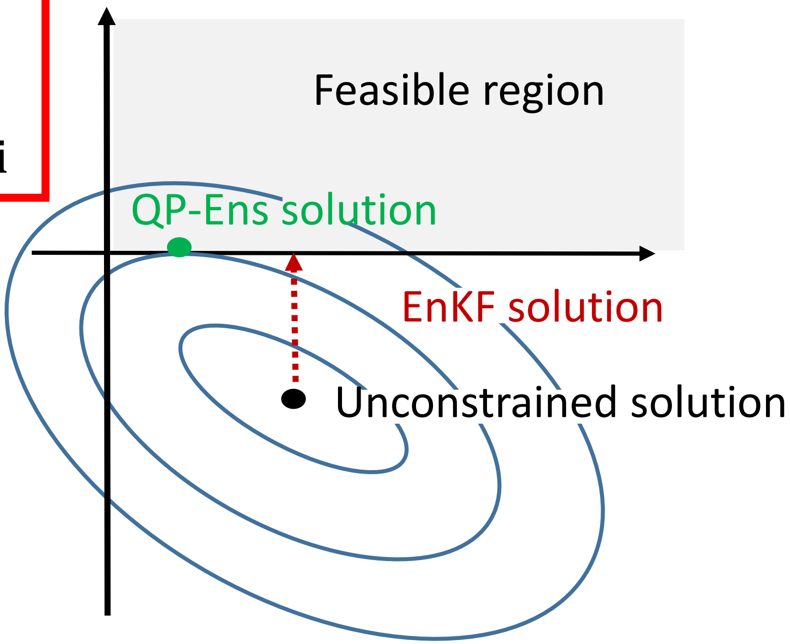
Algorithms

QPEns (Janjic et al, 2014)

$$x_i^a - x_i^f = \min_{\delta x} J(\delta x; v_i) \quad \text{subject to} \quad A\delta x \le a, \; B\delta x = b$$

EnKF (Evensen, 1994)

$$x_i^a - x_i^f = \min_{\delta x} J(\delta x; v_i) = PH^T(HPH^T + R)^{-1} v_i$$



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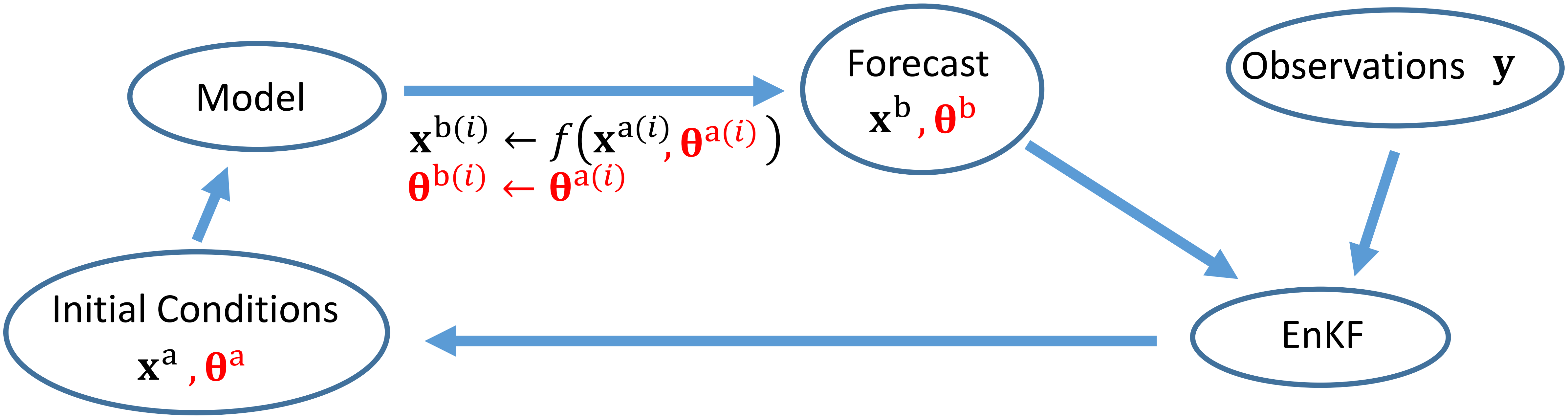
Augmented state parameter estimation



$$\begin{pmatrix} \bar{\mathbf{x}}^{a} \\ \bar{\boldsymbol{\theta}}^{a} \end{pmatrix} = \begin{pmatrix} \bar{\mathbf{x}}^{b} \\ \bar{\boldsymbol{\theta}}^{b} \end{pmatrix} + \begin{pmatrix} \mathbf{P} \\ \mathrm{Cov}(\boldsymbol{\theta}^{b}, \mathbf{x}^{b}) \end{pmatrix} \mathrm{H}^{T}\left(\mathrm{HPH}^{T} + \mathrm{R}\right)^{-1}(\mathbf{y} - \mathrm{H}\bar{\mathbf{x}}^{b})$$

Parameters are updated through their correlation with the state!

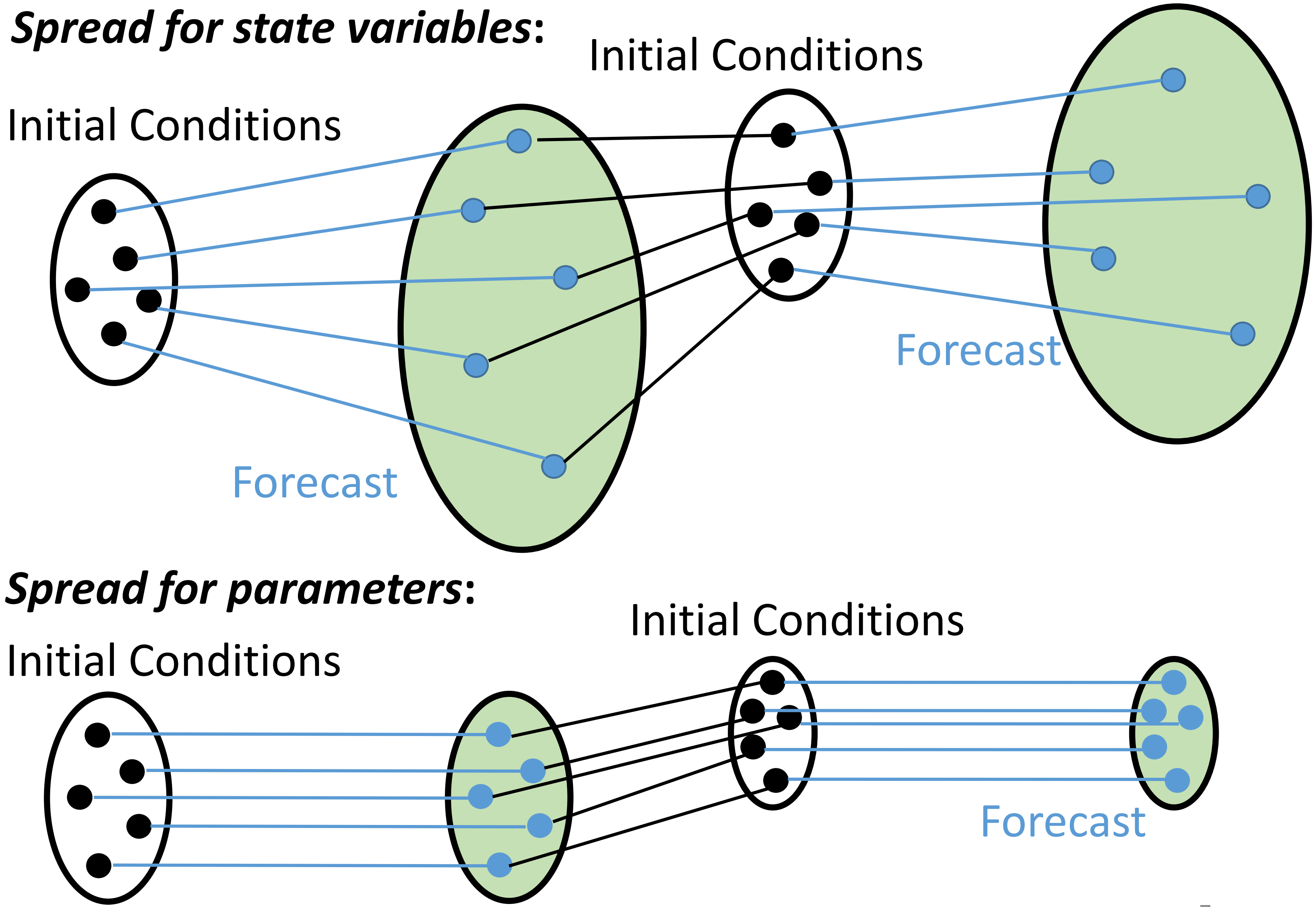
Dynamical model for parameters

$$\theta^f_{t,i} = \theta^a_{t-1,i} + f(\mathbf{X}_t)$$

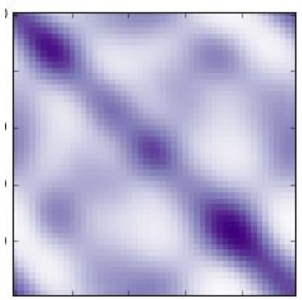
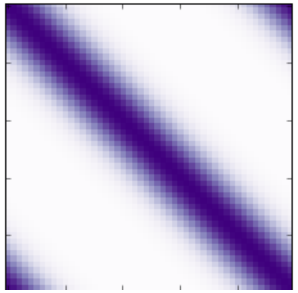
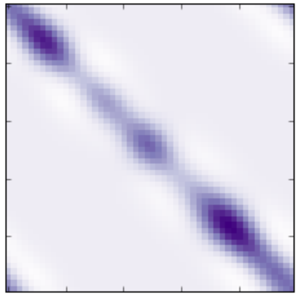
$$\mathbf{X}_t \sim \mathrm{Beta}(\alpha_t, \beta_t)$$

$$E[f(\mathbf{X}_t)] = \bar{\theta}^a_{t-1}$$

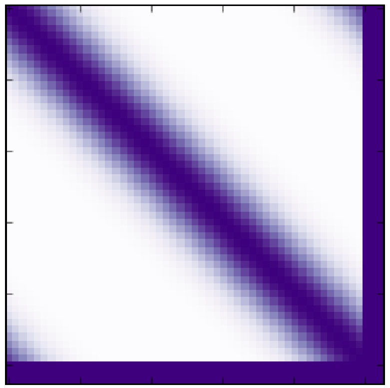
$$\mathrm{Var}\left[\theta^f_t\right] = \sigma^2$$



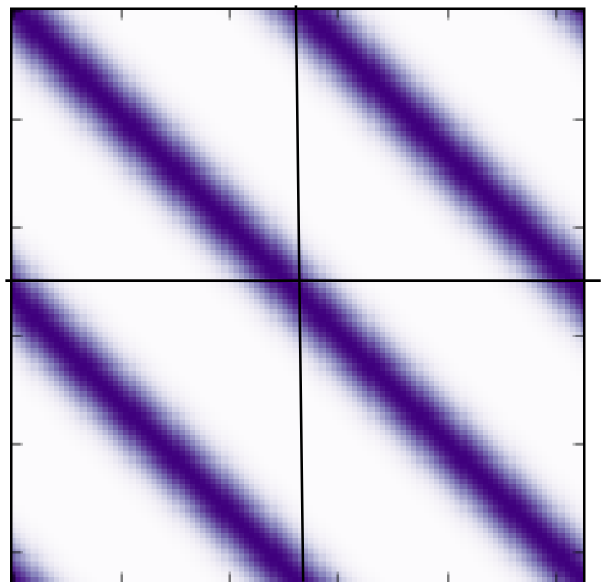
Covariance Localization (Gaspari and Cohn)

 $\circ$  $=$ 

No localization in parameter space → localization matrix positive indefinite!!



$$\begin{bmatrix} \mathbf{P} & \mathrm{Cov}(\mathbf{x}^{\mathrm{f}}, \boldsymbol{\theta}^{\mathrm{f}}) \\ \mathrm{Cov}(\mathbf{x}^{\mathrm{f}}, \boldsymbol{\theta}^{\mathrm{f}}) & \mathrm{Cov}(\boldsymbol{\theta}^{\mathrm{f}}, \boldsymbol{\theta}^{\mathrm{f}}) \end{bmatrix}$$



Localization matrix for the Quadratic Filter

$$\begin{bmatrix} \mathrm{P} & \mathrm{P}_{\mathrm{skew}} \\ \mathrm{P}_{\mathrm{skew}} & \mathrm{P}_{\mathrm{kurt}} \end{bmatrix}$$

Global updating:

$$\begin{bmatrix} \mathbf{L} & \mathbf{c} \\ \mathbf{c}^{\mathrm{T}} & 1 \end{bmatrix}$$ where $\mathbf{c} = \frac{1}{n}\mathbf{e}$

Experiment set-up

Modified Shallow Water Model
(Wuersch and Craig, 2014)

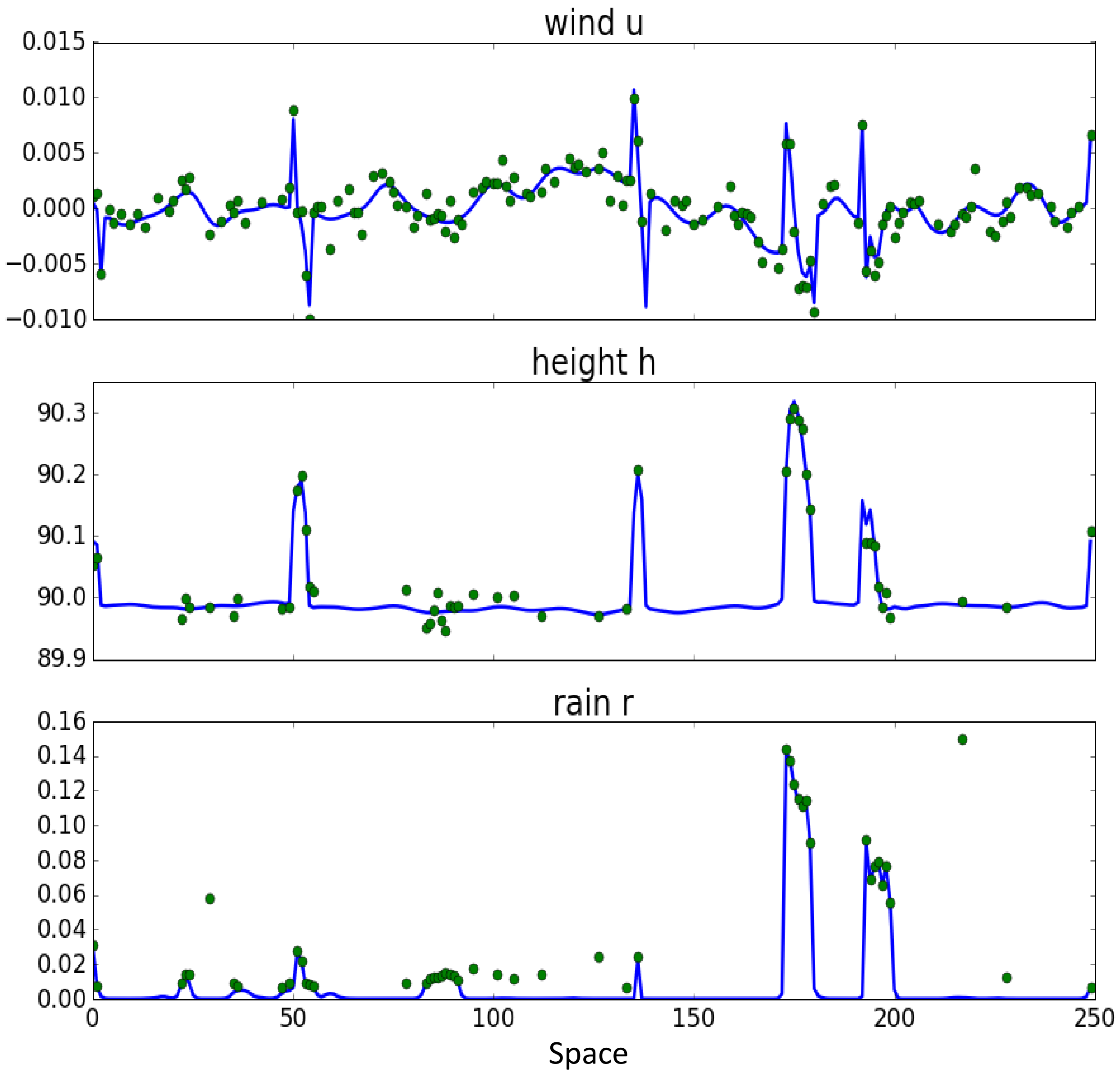
$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + \frac{\partial(\varphi + c^2 r)}{\partial x} = \beta_u + D_u \frac{\partial^2}{\partial x^2}$$

$$\frac{\partial h}{\partial t} + \frac{\partial(uh)}{\partial x} = D_h \frac{\partial^2 h}{\partial x^2}$$

$$\varphi = \begin{cases} \varphi_{c,} & if \;\; h > h_c \\ gh & otheriwse \end{cases}$$

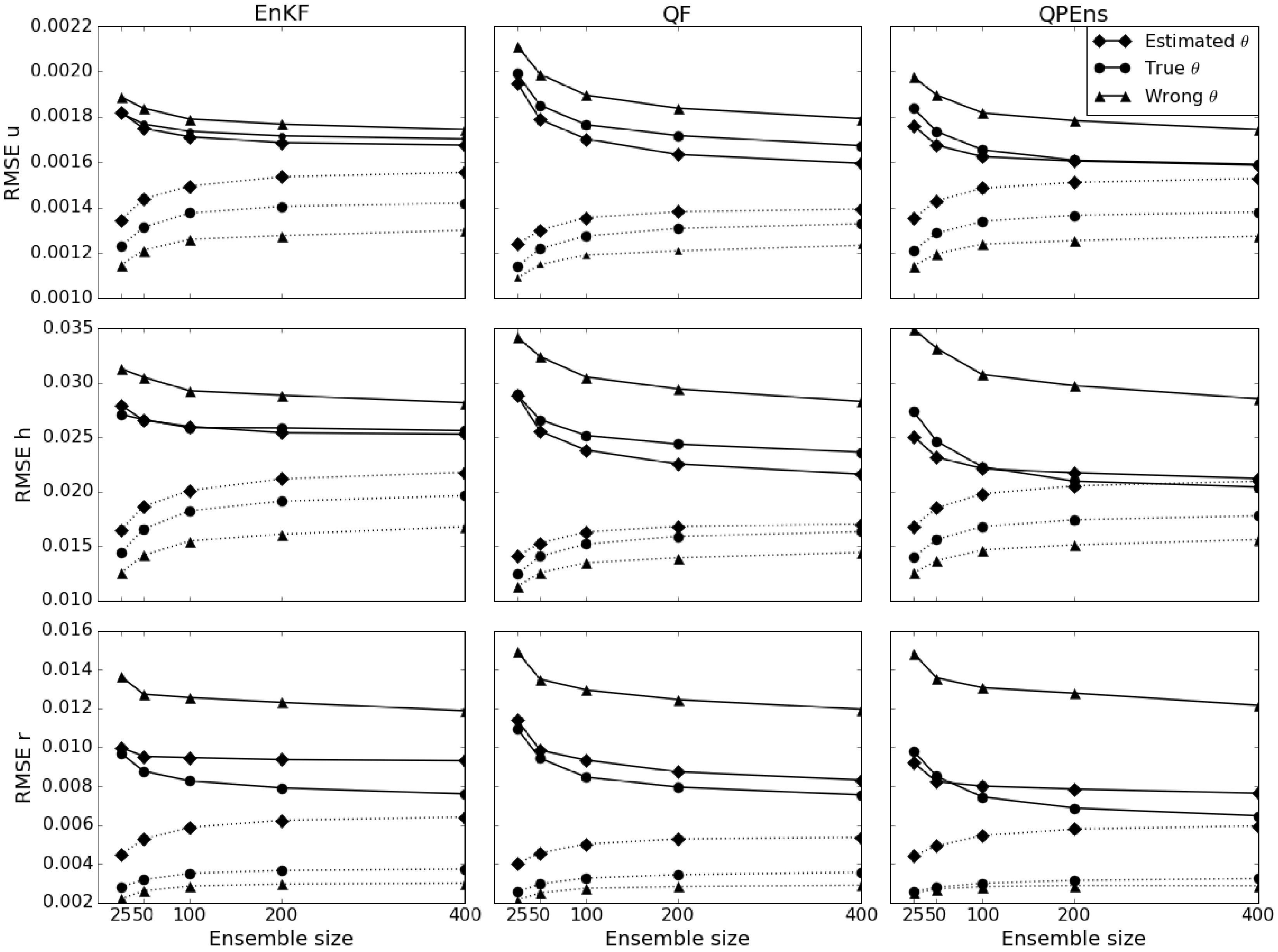
$$\frac{\partial r}{\partial t} + u\frac{\partial r}{\partial x} = D_r \frac{\partial^2 r}{\partial x^2} - \alpha r - \begin{cases} \delta \frac{\partial u}{\partial x}, & h > h_r \; and \; \frac{\partial u}{\partial x} < 0 \\ 0 & otherwise \end{cases}$$

- Twin experiment
- Radar and aircraft observations
- Initial parameter values are draw from uniform distributions

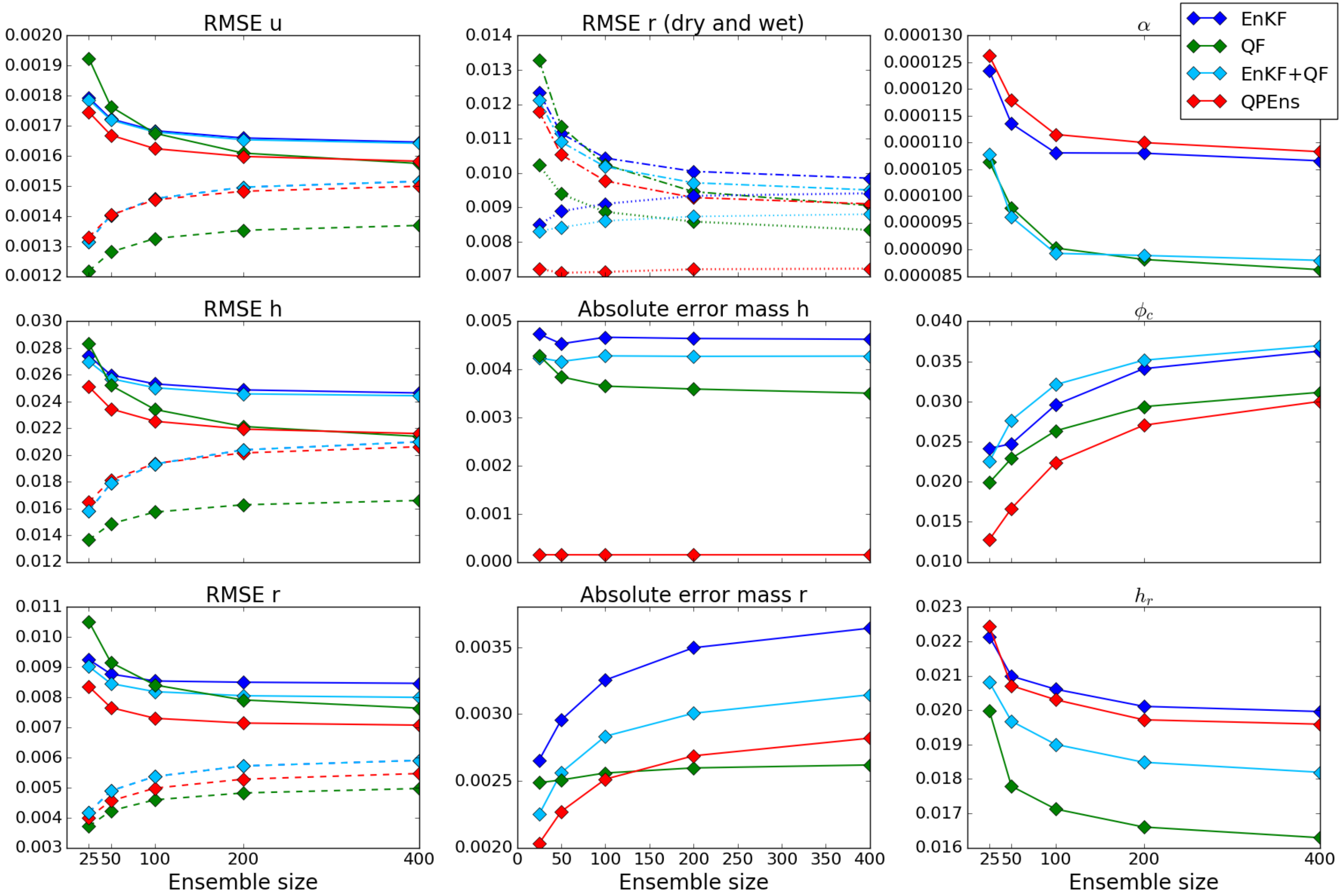


Results

- RMSE/Spread ratio is best when parameters are estimated for all algorithms
- QF is suited for parameter estimation

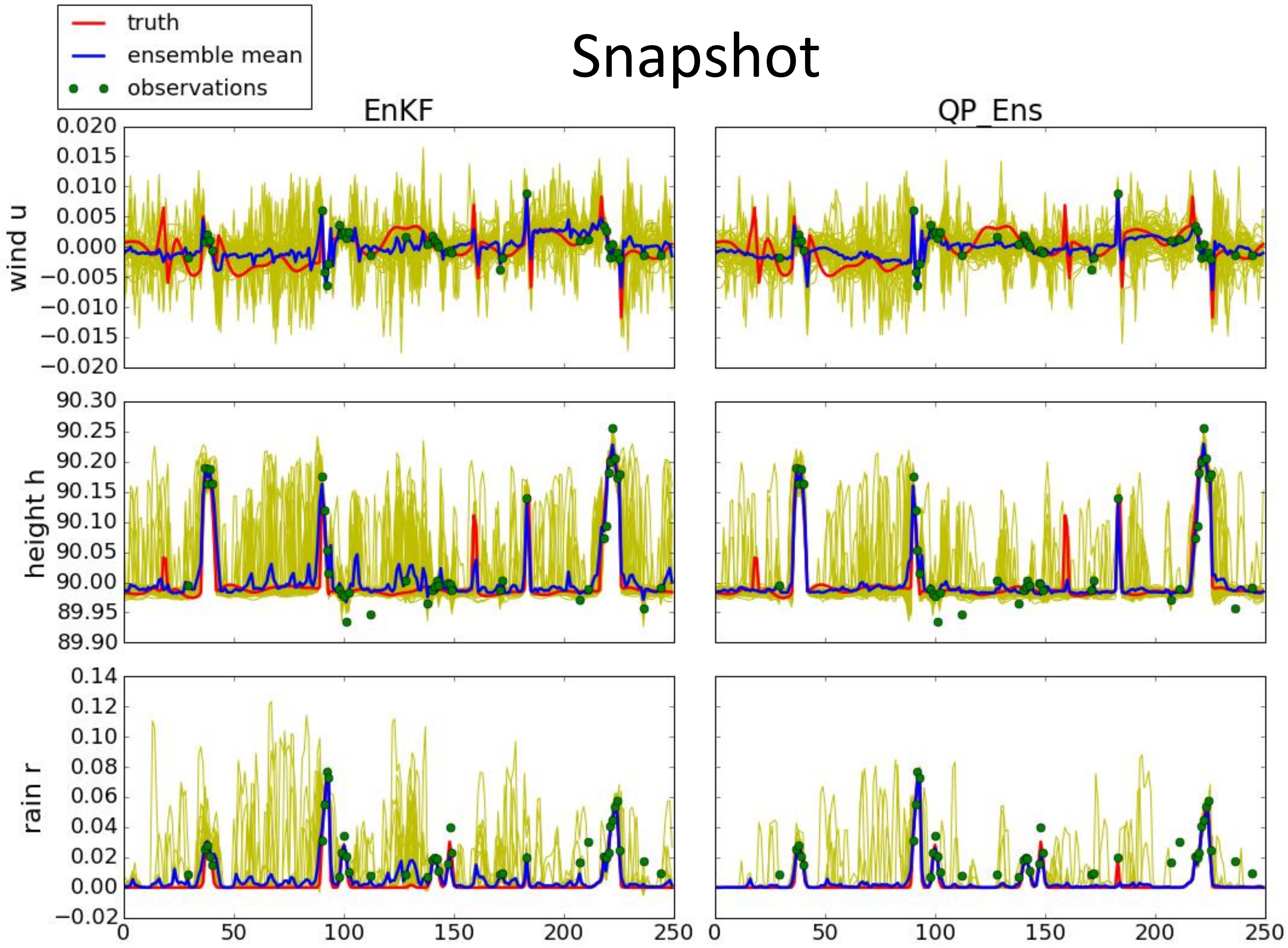


Algorithm comparison

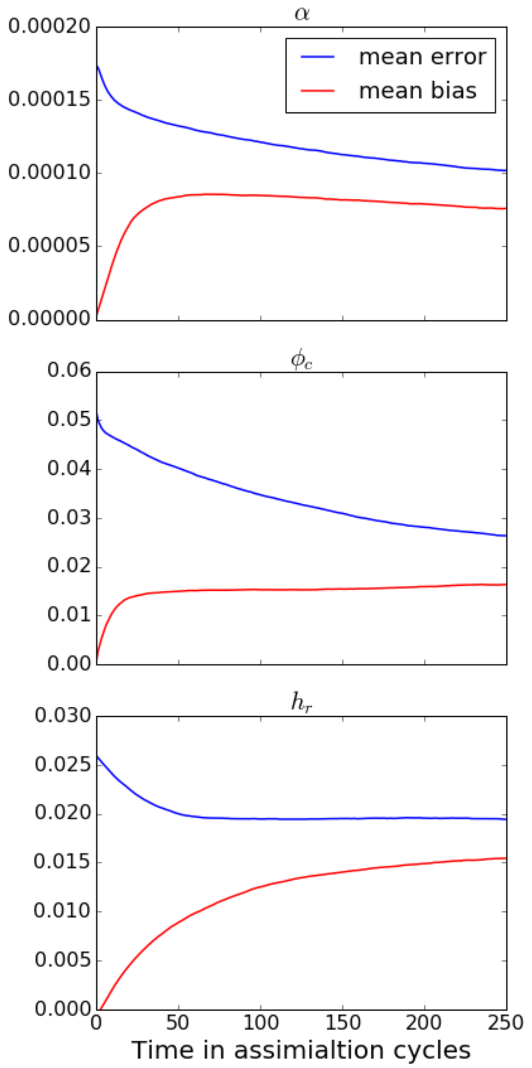


- QF needs a sufficiently large ensemble size to beat the EnKF but is most sensitive to ensemble size
- Positive feedback between state and parameters
- RMSE/Spread ratio is best for QPEns
- Taking higher order moments into account reduces mass bias
- QPEns suppresses spurious convection

Snapshot



Parameter error evolution for EnKF (50 members)



Conclusion

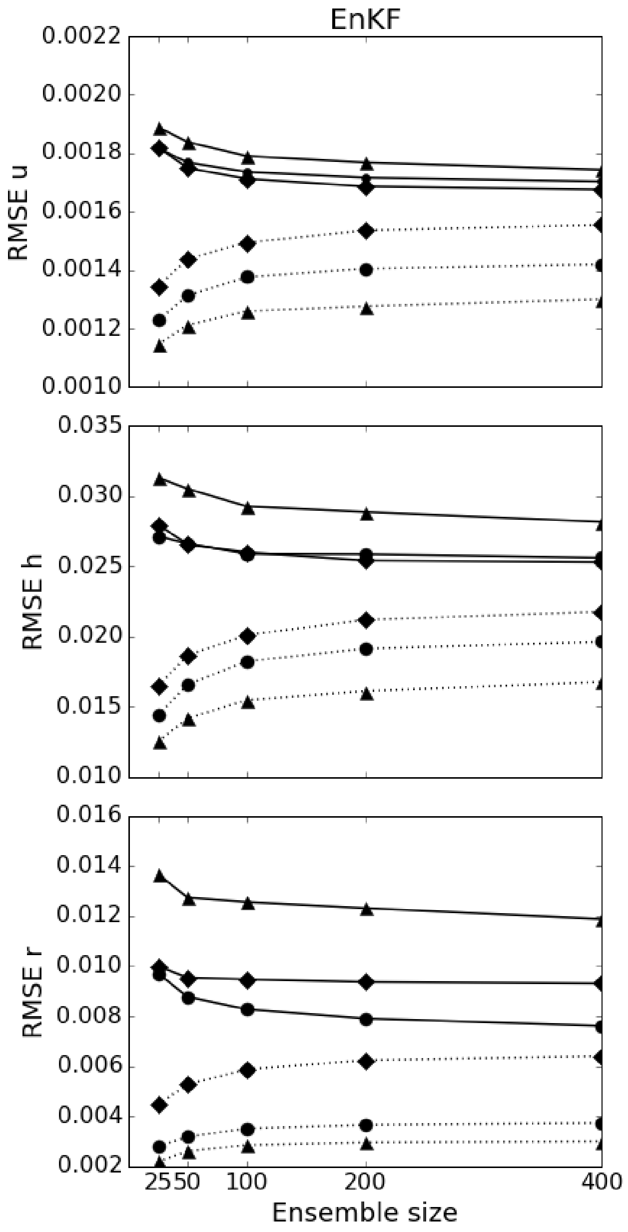
(Ruckstuhl and Janjic, 2018)

- For small ensemble sizes conserving physical properties is more beneficial
- Taking higher order moments into account becomes more beneficial as ensemble size increases
- Taking higher moments into account for parameter estimation shows benefits for all ensemble sizes

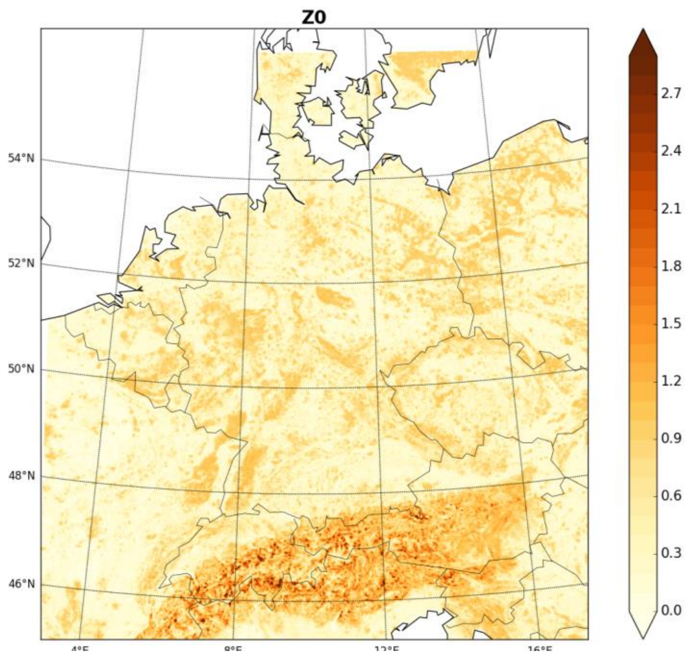
- Even EnKF does well for all ensemble sizes!!!

Outlook

- Estimation of roughness length in COSMO-KENDA

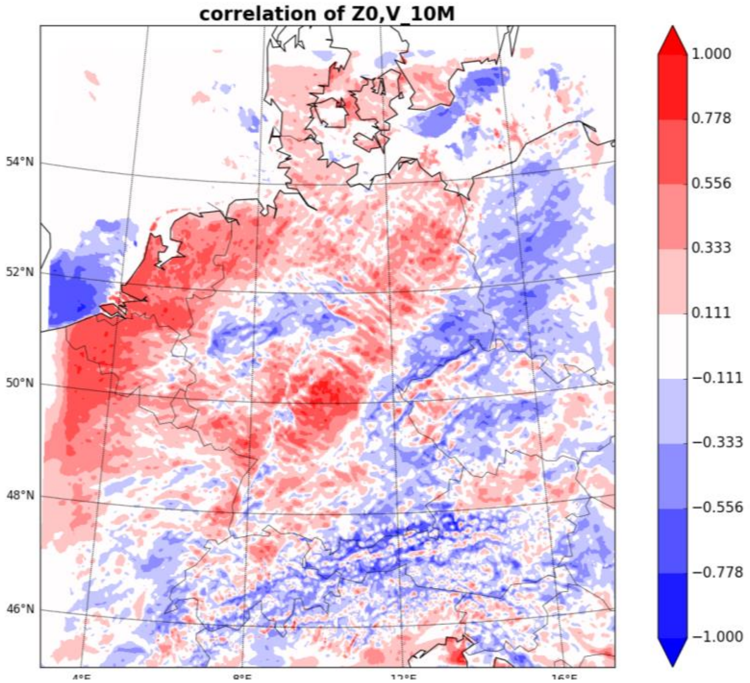


Estimation of roughness length in COSMO KENDA



Motivation

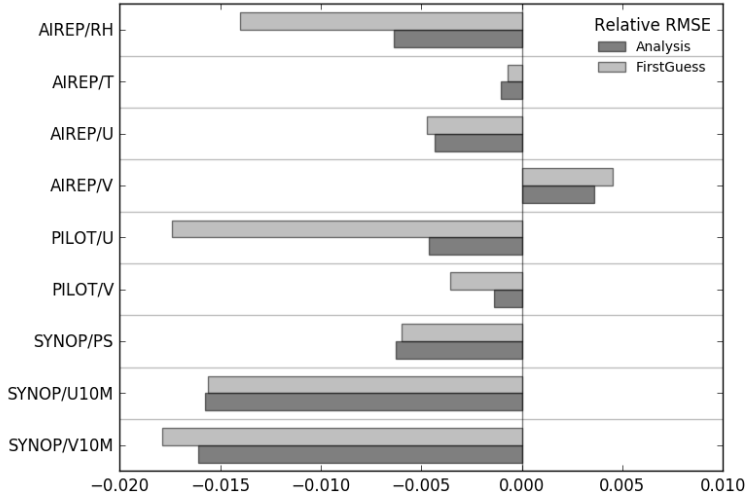
Cloud representation is highly dependent on accuracy of calculated surface fluxes. But calculation of surface fluxes contains many uncertainties. These uncertainties lead to model errors that are ignored in weather prediction systems.



Approach

Identify parameters that directly influence surface fluxes (roughness lengths) and estimate them along with the state to:

- Reduce forecast errors
- Better capture uncertainty of forecasts



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