# State-dependent Additive Covariance Inflation for Radar Reflectivity Assimilation

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# How to Assimilate Radar Reflectivity Directly

**Atmospheric State** (wind, temperature, pressure, humidity) — **Forecast** → **Hydrometeor** (cloud water, cloud ice, rain, snow, graupel)

**Cause**

**should be modified by reflectivity assimilation**

**Result**

**observed by radars**

To improve rainfall forecast by "direct" assimilation of radar reflectivity, atmospheric state should be modified based on **<u>correlation between the Atmospheric State and Hydrometeor</u>**

[3DVar] given climatologically (difficult to be estimated)

[4DVar] calculated by linear model (difficult to make the model)

[EnKF] calculated by ensemble forecasts (not difficult, but …)

# Problem of Reflectivity Assimilation with EnKF

Reflectivity assimilation with EnKF

**Analysis**

**First guess:** mean of ensemble forecasts ($u^f$, $v^f$, $w^f$, $T^f$, $p^f$, $q_v^f$, $q_c^f$, $q_{ci}^f$, $q_r^f$, $q_s^f$, $q_g^f$)

$$\mathbf{x}^a = \mathbf{x}^f + \mathbf{K}\left[ Z_H^{obs} - Z_H^f \right]$$

**Kalman gain:** weight of average between first guess and observation

$$\mathbf{K} = \mathbf{B}\mathbf{H}^T \left( \mathbf{H}\mathbf{B}\mathbf{H}^T + \mathbf{R} \right)^{-1}$$

Observed reflectivity (dBZ)

Forecasted reflectivity (dBZ) calculated from ($q_r^f$, $q_s^f$, $q_g^f$)

Obs. error covariance (given)

$$E\left[\delta\mathbf{x}^f \delta Z_H^f\right] \quad E\left[\delta Z_H^f \delta Z_H^f\right]$$

Forecast error covariance (calculated by ensemble forecast)

Problem: If rainfall is not forecasted, $\delta Z_H{}^f=0$ (no impact of $Z_H$ assimilation)

→ One solution: Inflation of $\delta Z_H{}^f$

# 3-type Error Covariance Inflation

$$\mathbf{K} = \mathbf{BH}^T\left(\mathbf{HBH}^T + \mathbf{R}\right)^{-1}$$

$$E\left[\delta\mathbf{x}^f \delta Z_H^f\right] \quad E\left[\delta Z_H^f \delta Z_H^f\right]$$

1. Multiplicative
(Anderson and Anderson 1999)

$$\boxed{\delta\mathbf{x}_i^f \leftarrow \rho\delta\mathbf{x}_i^f} \quad (\rho > 1)$$

→ If $\delta Z_H^f$=0, $\mathbf{BH}^T$=0

2. Additive
(Mitchell and Houtekamer 2000)

$$\boxed{\delta\mathbf{x}_i^{f(a)} \leftarrow \delta\mathbf{x}_i^{f(a)} + \mathbf{q}_i} \quad \left(\mathbf{q}_i \sim N\left(0, \sigma^2\right)\right)$$

→ If $\delta Z_H^f$=0, $\mathbf{BH}^T=E[\delta\mathbf{x}_i^f\mathbf{q}_i]$=0
because $\mathbf{q}_i$ is random perturbation

3. Relaxation to prior
(Zhang et al. 2004)

$$\boxed{\delta\mathbf{x}_i^a \leftarrow \alpha\delta\mathbf{x}_i^f + (1-\alpha)\delta\mathbf{x}_i^a} \quad (0 < \alpha < 1)$$

→ If $\delta Z_H^f$=0, $\mathbf{BH}^T$=0

→ We propose to introduce
**non-random $\delta Z_H$ that has reasonable correlation with $\delta\mathbf{x}^f$**

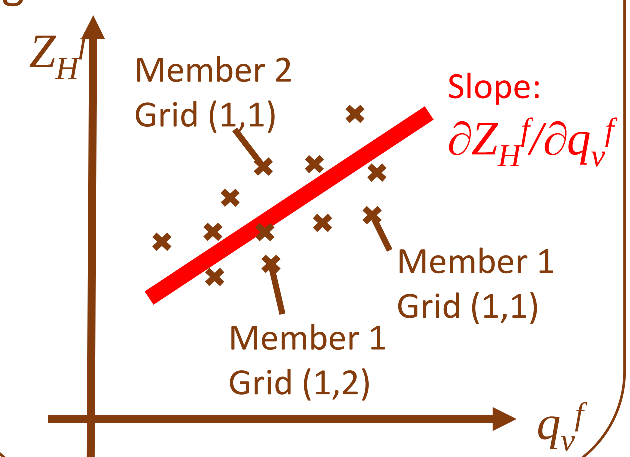
# State-dependent Additive Inflation of $Z_H^f$

$Z_H^f$ of member $i$ is replaced with following $\delta Z_H^{f(i)}$ before assimilation if rainfall is not forecasted at obs. points in all members

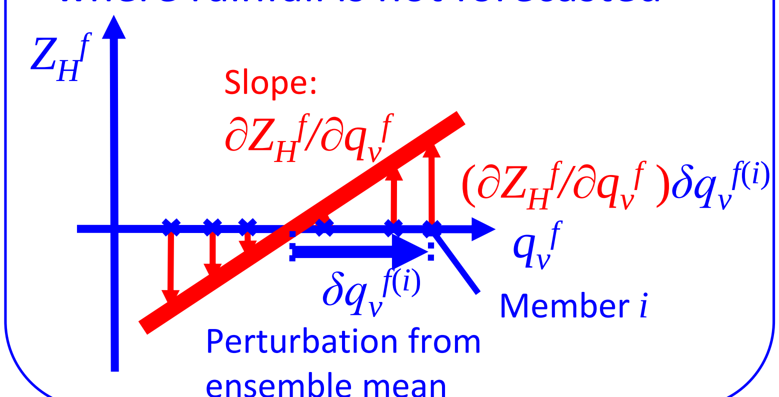
zonal wind — meridional wind — vertical wind — temperature — water vapor mixing ratio

$$\delta Z_H^{f(i)} = \frac{\partial Z_H^f}{\partial u^f}\delta u^{f(i)} + \frac{\partial Z_H^f}{\partial v^f}\delta v^{f(i)} + \frac{\partial Z_H^f}{\partial w^f}\delta w^{f(i)} + \frac{\partial Z_H^f}{\partial T^f}\delta T^{f(i)} + \boxed{\frac{\partial Z_H^f}{\partial q_v^f}\delta q_v^{f(i)}}$$

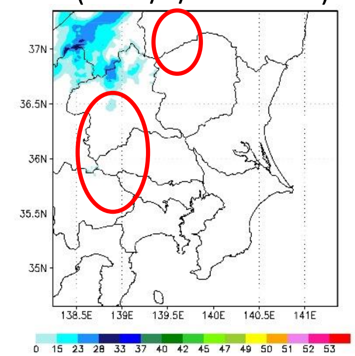
Slope of regression line of $q_v^f$ for $Z_H^f$ calculated with all grids in each vertical level

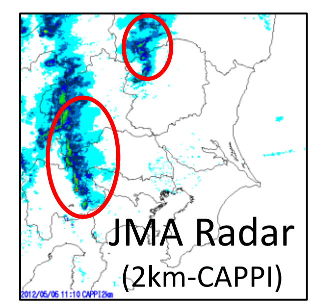



This term is calculated in points where rainfall is not forecasted




$Z_H^f$ @z*=2km
(2012/5/6 1110JST)



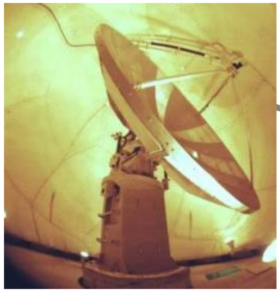

→ Additional $\delta Z_H^f$ is not random but correlated with $(u^f, v^f, w^f, T^f, q_v^f)$

→ Atmospheric state can be modified by $Z_H$ assimilation

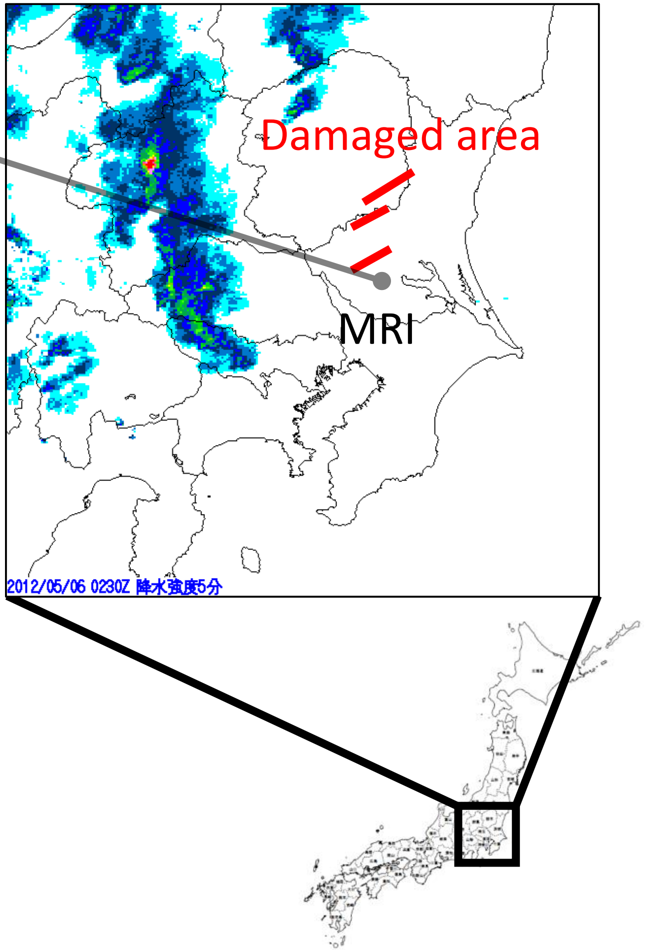
# Target: Tornadic Supercell in 6 May 2012

3 tornadoes were generated almost simultaneously at 1230 JST.
South one is estimated F3 (70-92 m/s).
There were dense radar and surface observations.

MRI advanced C-band solid-state polarimetric (MACS-POL) radar



JMA radar



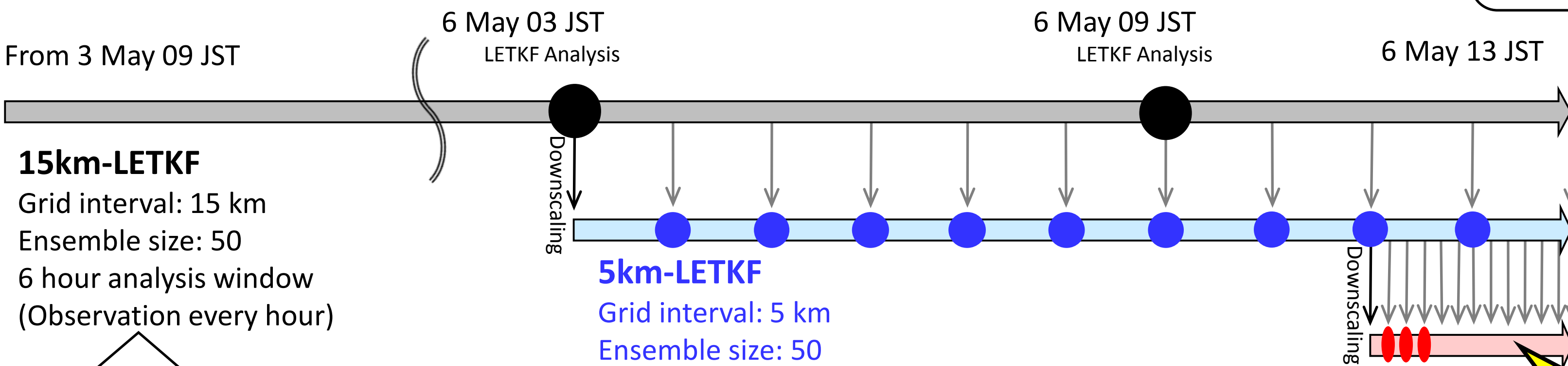

**→ Development of the tornadic supercell was observed in detail by MACS-POL**

**→ Suitable for investigating assimilation impact of the polarimetric radar**

Experiment to Check Impact of $\delta Z_H{}^f$

# Assimilation Experiments with LETKF

Local ensemble transform Kalman filter




**15km-LETKF**
Grid interval: 15 km
Ensemble size: 50
6 hour analysis window
(Observation every hour)

Hourly operational obs. are assimilated every 6 hours

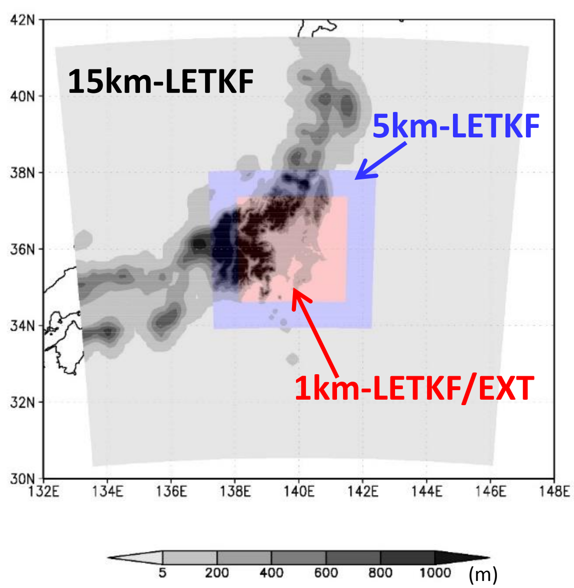
**5km-LETKF**
Grid interval: 5 km
Ensemble size: 50
1 hour analysis window
(Observation every 10 minutes)

10-min radar (radial wind) and surface obs. are assimilated every 1 hour

**1km-LETKF/EXT**
Grid interval: 1 km
Ensemble size: 50
10 minutes analysis window
(Observation every minute)

**Extended forecasts from 1130JST**

1-min radar (radial wind and **reflectivity of MACS-POL**) and surface obs. are assimilated every 10 minutes




: Ensemble forecasts
: LETKF analysis
: Abbreviation of 6-hour forecast-analysis cycle

Operational obs.: Surface (pressure), Radiosondes (wind / temperature / humidity), Aircrafts (wind / temperature), Radars (Doppler wind / humidity), Wind profiler radars (wind), Microwave scatterometers (wind), Visible / infrared imagers (wind), and GNSS (precipitable water vapor)

# How to Assimilate $Z_H^{obs}$

- $Z_H^f$[dBZ] is calculated from forecasted rain, snow, and graupel ($q_r^f$, $q_s^f$, $q_g^f$)

$$Z_H^f\left(q_r^f,q_s^f,q_g^f\right)=10\log_{10}\left[\underbrace{720N_{0r}\left(\frac{\rho q_r^f}{\pi\rho_r N_{0r}}\right)^{7/4}}_{\text{rain}}+\underbrace{720N_{0s}\left(\frac{\rho q_s^f}{\pi\rho_s N_{0s}}\right)^{7/4}\times\boxed{0.23\frac{\rho_s^2}{\rho_r^2}}}_{\text{snow}}+\underbrace{720N_{0g}\left(\frac{\rho q_g^f}{\pi\rho_g N_{0g}}\right)^{7/4}\times0.23\frac{\rho_g^2}{\rho_r^2}}_{\text{graupel}}\right]$$

Set to be 1 if $T\geq 0℃$ (bright band)

Theoretical formulation in 1-moment cloud microphysics

$$N(D)=N_0e^{-\Lambda D}$$

cf., Dowell et al. (2011)

Number concentration

- rain $N_{0r}=8\times10^6[\mathrm{m}^{-4}]$
- snow $N_{0s}=1.8\times10^6[\mathrm{m}^{-4}]$
- graupel $N_{0g}=1.1\times10^6[\mathrm{m}^{-4}]$

Mass density

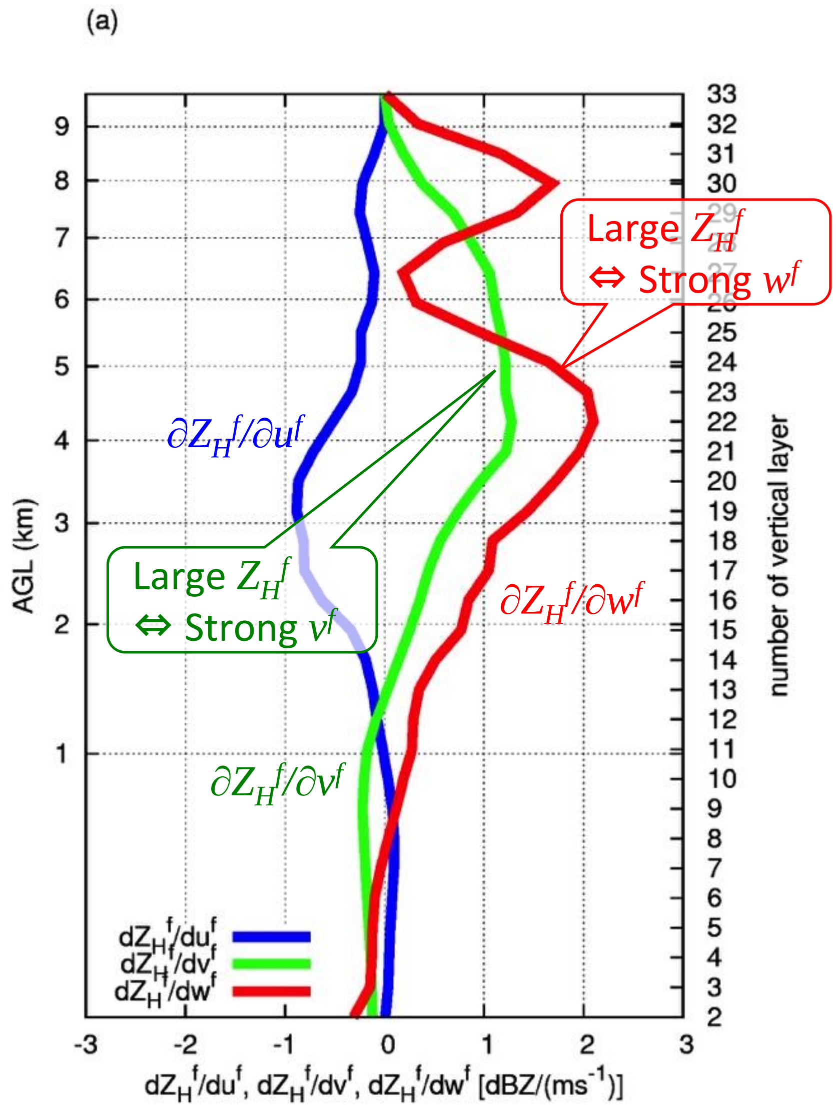
- rain $\rho_r=1\times10^3[\mathrm{kg\,m}^{-3}]$
- snow $\rho_s=84[\mathrm{kg\,m}^{-3}]$
- graupel $\rho_g=3\times10^2[\mathrm{kg\,m}^{-3}]$

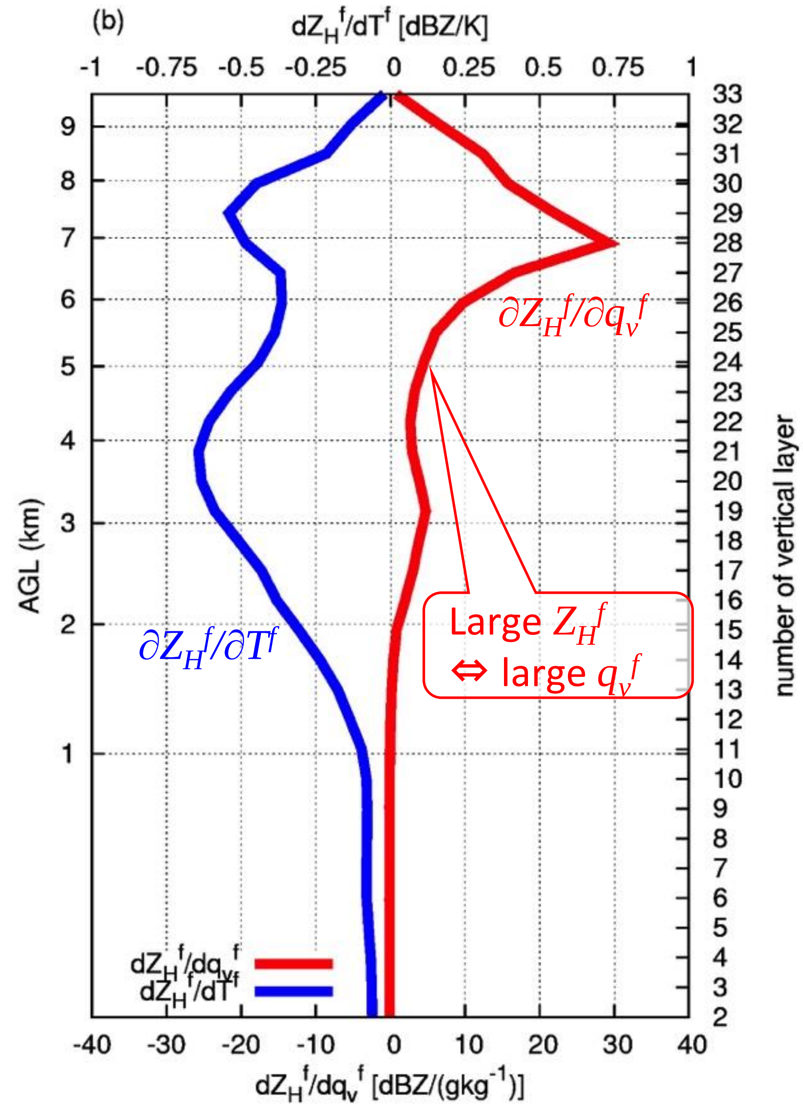
- Observation error variance of $Z_H^{obs}$: $\sigma_Z^2=(5\mathrm{dBZ})^2$
- Attenuation of $Z_H^{obs}$ is corrected by $Z_H^{obs}+0.073\Phi_{DP}^{obs}\rightarrow Z_H^{obs}$ (Jameson 1992)
- $Z_H^{obs}$ is interpolated to 2-km grid (influence radius: 1km) before assimilation
- $Z_H^{obs}$=0dBZ [$\sigma_Z^2=(50\mathrm{dBZ})^2$] is assimilated at points of $Z_H^{obs}$<15dBZ

# Characteristic of Added $\delta Z_H^f$

Slope of regression line ($\delta Z_H^f/\delta \mathbf{x}^f$) in 1110JST

$$\delta Z_H^{f(i)} = \frac{\partial Z_H^f}{\partial u^f}\delta u^{f(i)} + \frac{\partial Z_H^f}{\partial v^f}\delta v^{f(i)} + \frac{\partial Z_H^f}{\partial w^f}\delta w^{f(i)} + \frac{\partial Z_H^f}{\partial T^f}\delta T^{f(i)} + \frac{\partial Z_H^f}{\partial q_v^f}\delta q_v^{f(i)}$$



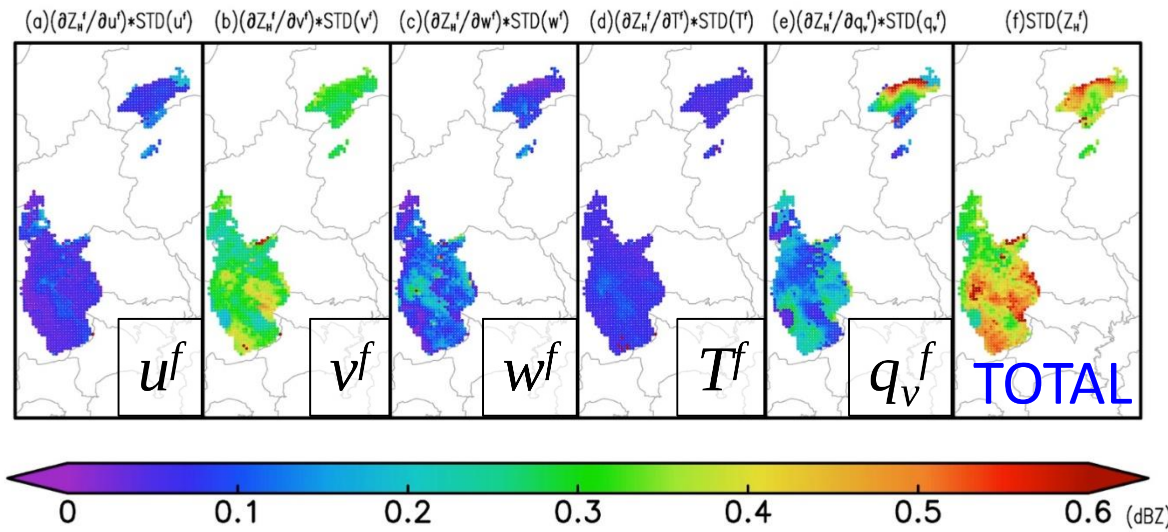



# Characteristic of Added $\delta Z_H^{\,f}$

$$\delta Z_H^{\,f(i)} = \frac{\partial Z_H^{\,f}}{\partial u^f}\delta u^{f(i)} + \frac{\partial Z_H^{\,f}}{\partial v^f}\delta v^{f(i)} + \frac{\partial Z_H^{\,f}}{\partial w^f}\delta w^{f(i)} + \frac{\partial Z_H^{\,f}}{\partial T^f}\delta T^{f(i)} + \frac{\partial Z_H^{\,f}}{\partial q_v^{\,f}}\delta q_v^{\,f(i)}$$

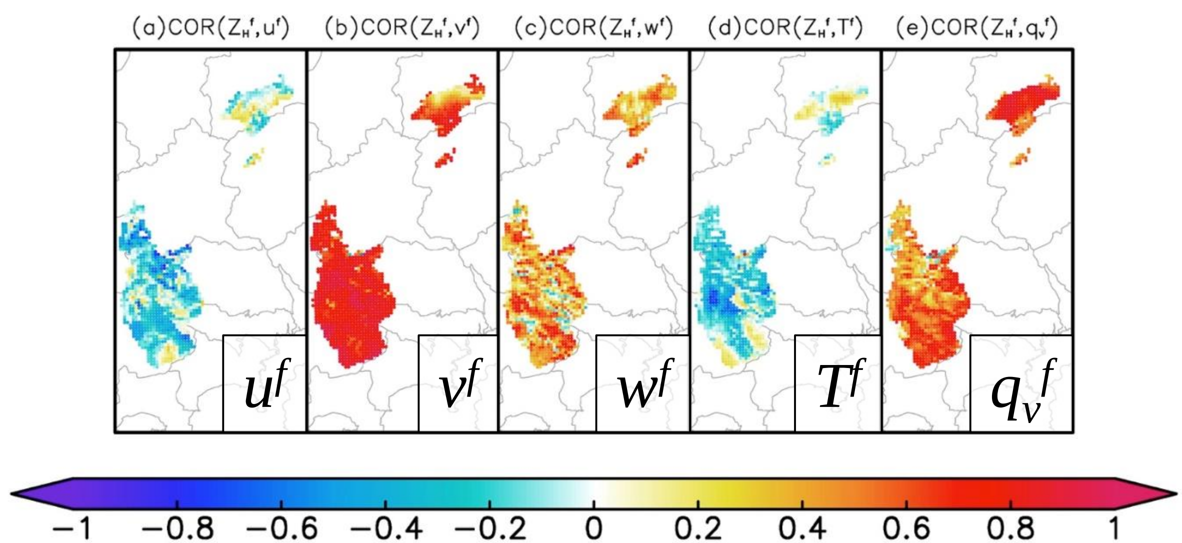
Each term of standard deviation of $\delta Z_H^{\,f(i)}$ (dBZ) in 1110JST @5-km AGL

Contributions of $\delta v^f$ and $\delta q_v^{\,f}$ are larger




Correlation between $\mathbf{x}^f=(u^f, v^f, w^f, T^f, q_v^{\,f})$ and $\delta Z_H^{\,f}$ in 1110JST @5-km AGL

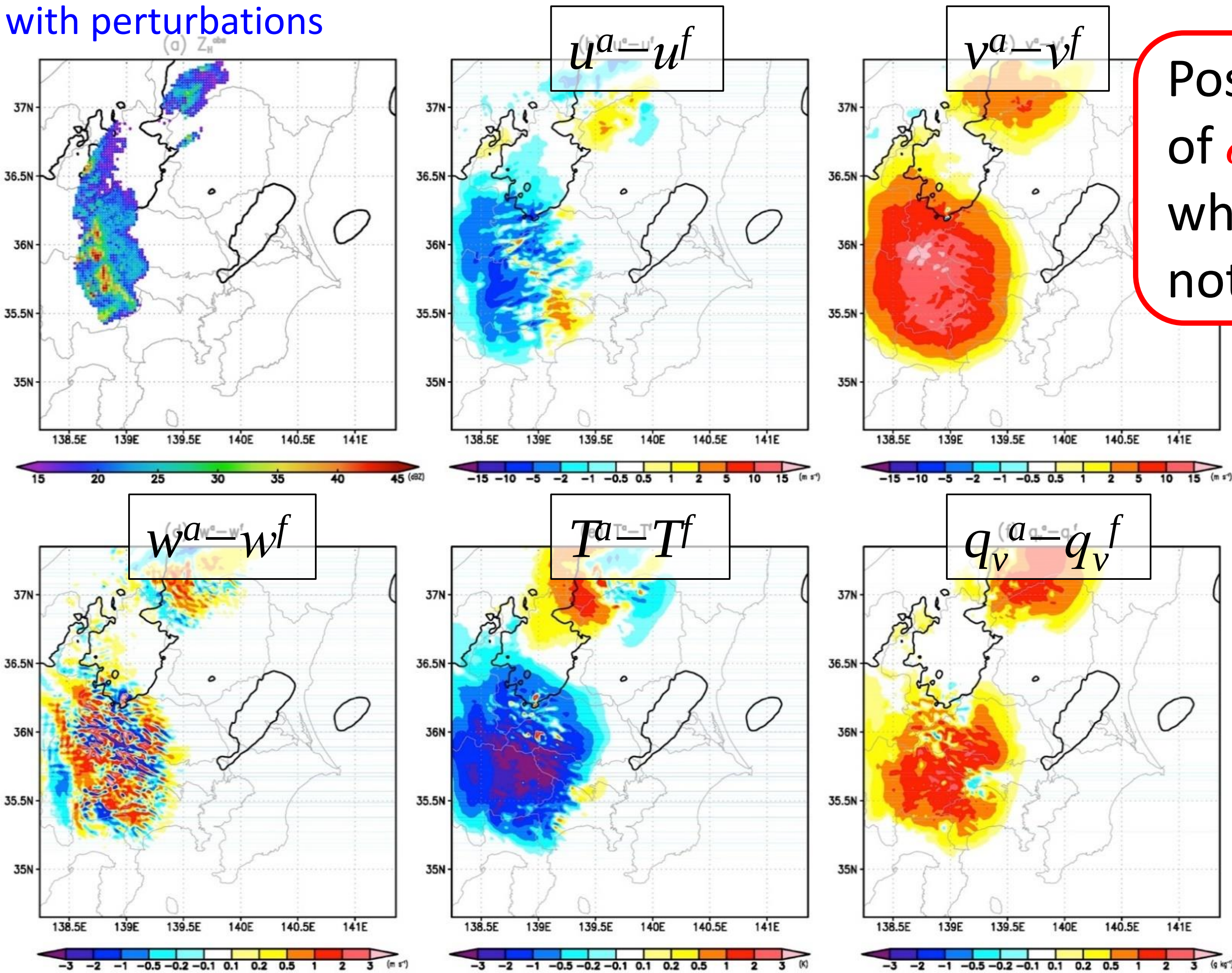
Large correlation of $\delta v^f$ and $\delta q_v^{\,f}$



# Modification of Atmospheric State by Additional $\delta Z_H^f$

$Z_H^{obs}$ assimilated with perturbations

Analysis increment ($\mathbf{x}^a - \mathbf{x}^f$) in 1110JST @5-km AGL (Contour shows $Z_H^f$=0dBZ)




Positive increment of $\delta v^f$ and $\delta q_v^f$ where rainfall is not forecasted

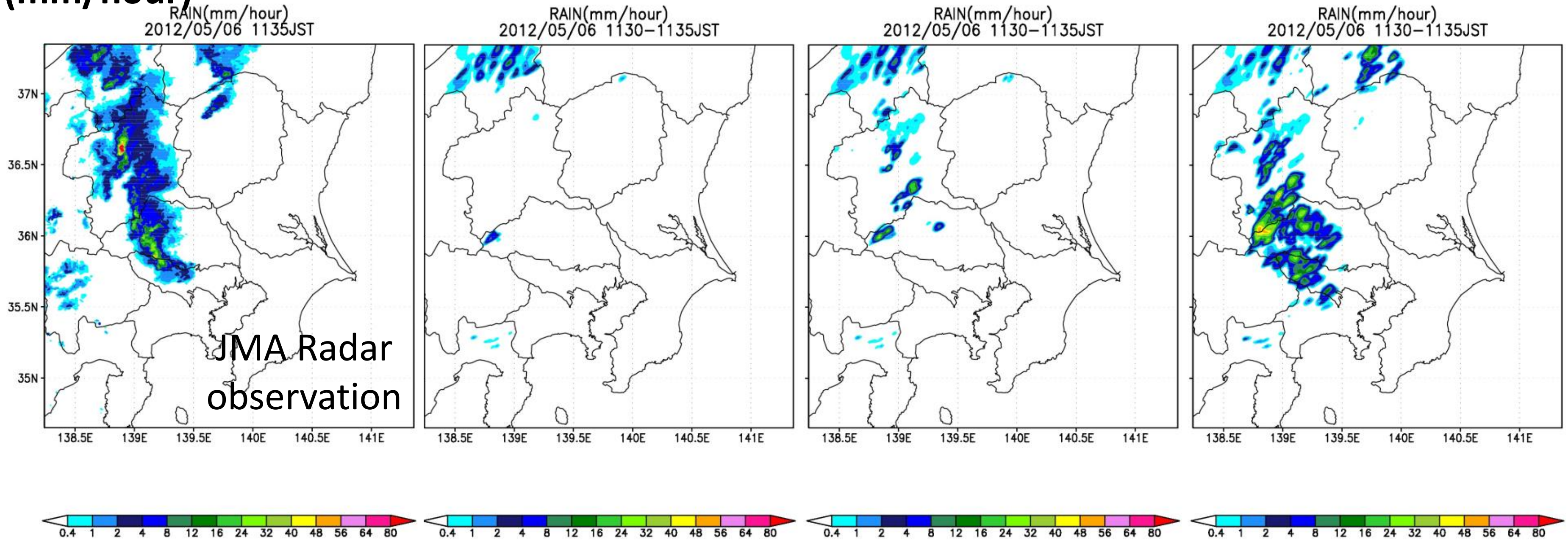
# Change of Rainfall Forecast by Additional $\delta Z_H^f$

**Rainfall in 1130-1300JST (mm/hour)**

Not assimilate $Z_H^{obs}$

Assimilate $Z_H^{obs}$ without additional $\delta Z_H^f$

Assimilate $Z_H^{obs}$ with additional $\delta Z_H^f$



Results (2012/5/6)



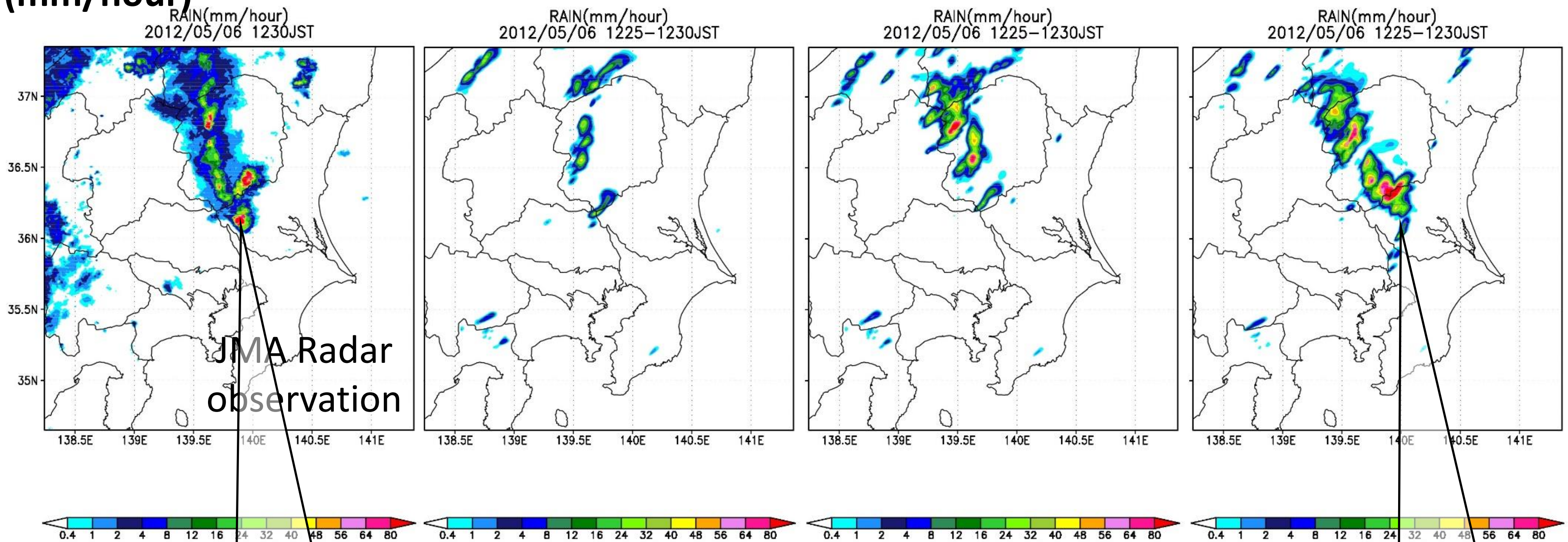
# Change of Rainfall Forecast by Additional $\delta Z_H^f$

**Rainfall in 1225-1230JST (mm/hour)**

Not assimilate $Z_H^{obs}$

Assimilate $Z_H^{obs}$ without additional $\delta Z_H^f$

Assimilate $Z_H^{obs}$ with additional $\delta Z_H^f$




F3 tornado was generated here at this time

Strong rainfall was successfully forecasted

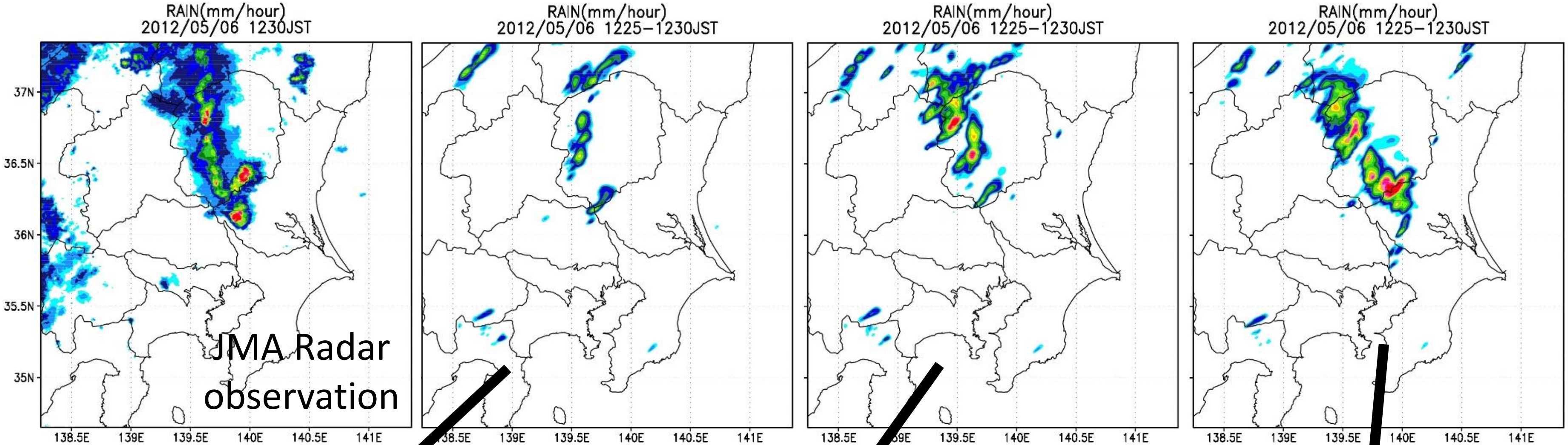
# Fractions Skill Score Verification

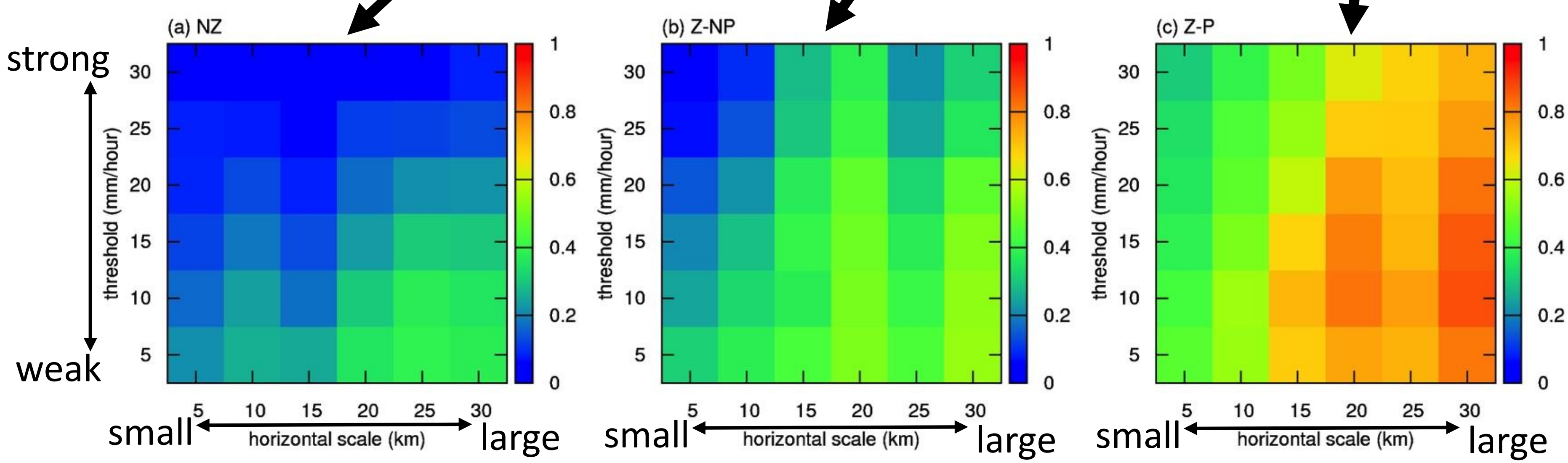
**To compare rainfall estimated by JMA radars at 1230JST**

Not assimilate $Z_H^{obs}$

Assimilate $Z_H^{obs}$ without additional $\delta Z_H^f$

Assimilate $Z_H^{obs}$ with additional $\delta Z_H^f$



# Fractions Skill Score Verification

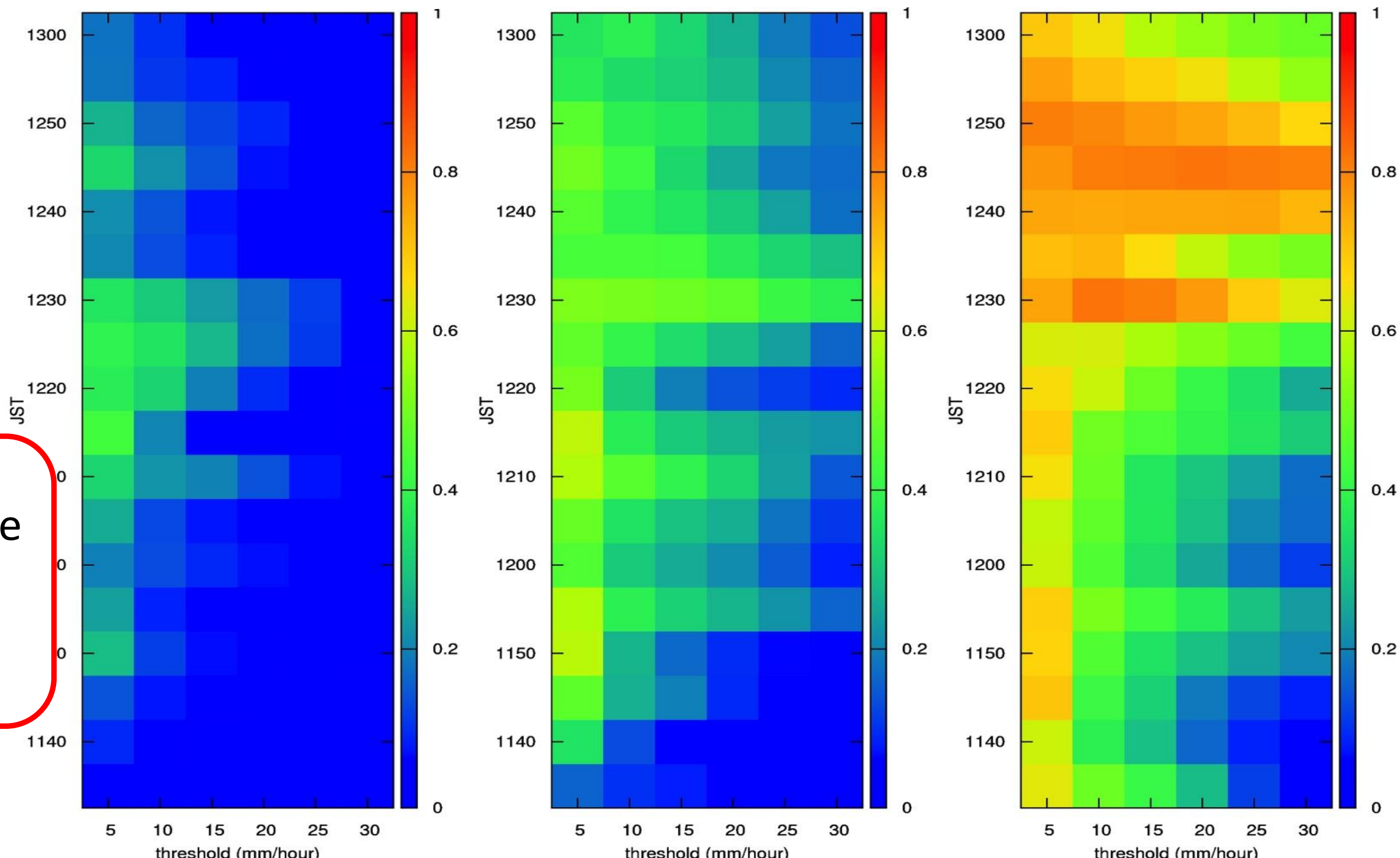
**Verification using rainfall estimated by JMA radars between 1130-1300JST**

**(horizontal scale=20km)**

Not assimilate $Z_H^{obs}$

Assimilate $Z_H^{obs}$ without additional $\delta Z_H^f$

Assimilate $Z_H^{obs}$ with additional $\delta Z_H^f$




weak ⟷ strong

Improvement starts from the initial time and remains for 90 min.

# Summary

- **Summary**
  - To improve rainfall forecast by "direct" assimilation of $Z_H$, atmospheric state should be modified based on **correlation between the Atmospheric State and Hydrometeor**
  - However, there is no impact of assimilation of $Z_H$ at points where rainfall is not forecasted

  → We suggest to **add $Z_H$ perturbations correlated with the atmospheric state** before assimilation at points where rainfall was not forecasted. (It has possibly to improve short-term rainfall forecast.)

- **Future issues**
  - Applying to other cases (local rainfall, snow, and so on)
  - Improving making perturbations when the number of samples is small
  - Improving attenuation correction of snow and graupel

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