Direct Variational Assimilation of Radar Reflectivity and Radial Velocity Data: Issues with Nonlinear Reflectivity Operator and Solutions

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Liu, Xue and Kong (2018, to be submitted)

Outline

- Background and motivations
- Reflectivity observation operator
- Problems and treatments associated with the reflectivity operator
- Experimental results
- Summary

Previous studies of Z assimilation

- Cloud analysis (Albers et al. 1996; Xue et al. 2003; Hu et al. 2006; Kain et al. 2010; Sun et al. 2014)
  - Relatively cheaper computational cost and easier implementation
  - Rely on empirical relationship
  - Does not use error statistics

- EnKF (Aksoy et al. 2009, 2010; Jung et al. 2008a; Jung et al. 2008b; Tong and Xue 2005; Xue et al. 2006)
  - Does not use TL and Adj.
  - Can include nonlinear operator but the solution is suboptimal (Lorenc 2003)
  - Affected by Sampling error

Previous studies of Z assimilation

- Variational
  - 4DVar (Sun and Crook 1997)
    - Compared direct assimilation of Z and assimilation of derived rainwater
    - Considered warm rain only
  - 3DVar (Xiao et. al. 2007; Gao and Stensrud 2012)
    - Has low computational cost
    - Uses flow-independent **B**
  - Hybrid EnVar (Carley 2012; Wang and Wang 2017; Kong et al. 2018)
    - Using logarithmic mixing ratios as the control variables (Carley 2012)
    - a pure EnVar method without using the TL and ADJ Z operator (Wang and Wang 2017)
    - Compared direct Z assimilation using hybrid EnVar and EnKF (Kong et al. 2018)

Motivations

- Aim to further improve direct Z DA based on the variational framework
  - Study the problems with the nonlinear Z operator when considering Multi-phased MP
  - Propose some treatments to deal with the problems
  - Examine the impacts of the treatments through OSSEs.

Z operator considering multi-phased hydrometeors (take Lin scheme as example)

$$Z = 10\log Z_e \qquad Z_e = Z_{er} + Z_{es} + Z_{eh}$$

$$Z_{er} = 3.63\times10^{9}\times\left(\rho q_r\right)^{1.75} \quad Z_{es} = \begin{cases} 9.80\times10^{8}\times\left(\rho q_s\right)^{1.75} & T_b \le 0^\circ \\ 4.26\times10^{11}\times\left(\rho q_s\right)^{1.75} & T_b > 0^\circ \end{cases} \quad Z_{eh} = 4.33\times10^{10}\times\left(\rho q_h\right)^{1.75}$$

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- Background and motivations
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- Problems and treatments with the reflectivity operator
  - Using mixing ratio as control variable (CVq)
  - Using logarithmic mixing ratio as control variable (CVlogq)
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Problems with CVq

$$\nabla J = \mathbf{B}^{-1}(\mathbf{x} - \mathbf{x}_b) + \mathbf{H}^{\mathrm{T}}\mathbf{R}^{-1}(H(\mathbf{x}) - \mathbf{y})$$

Single-phase TL: $\mathrm{H} = \dfrac{\partial Z}{\partial q_r} = \dfrac{17.5}{\ln 10 \times q_r}$

Multi-phase TL: $\mathrm{H} = \dfrac{\partial Z}{\partial q_r} = \dfrac{6.35 \times 10^{10} \times \rho^{1.75} q_r^{\ 0.75}}{\ln 10 \times Z_e}$

- For single-phased hydrometeor, $q_r \Rightarrow 0, \partial Z / \partial q_r \Rightarrow \infty$

  a lower limit for *q* is used (Sun and Crook 1997, Wang et al 2013, Wang and Wang 2017).

- For multi-phased hydrometeors, $Z_e \Rightarrow 0, \partial Z / \partial q_r \Rightarrow \infty$

  a new treatment of modified Z_e (instead of q) is proposed

Special Treatments for CVq

- Old treatment:

  adding a lower limit to the mixing ratios (qLim, Sun and Crook 1997)

$$Z = H(\max(q, \varepsilon_q)) \qquad \frac{\partial Z}{\partial q} = \begin{cases} \dfrac{\partial Z(q)}{\partial q} & q > \varepsilon_q \\ 0 & q \le \varepsilon_q \end{cases}$$

*Limitation: Reflectivity observations are ignored when $q \le \varepsilon_q$, due to zero TL term.

- New treatments:

1. Modify Z_e if smaller than ε_{Z_e} (ZeLim)

$$Z_e = \begin{cases} Z_e(q) & Z_e >= \varepsilon_{Z_e} \\ Z_e(q) + \varepsilon_{Z_e} & Z_e < \varepsilon_{Z_e} \end{cases}$$

2. Use a separate pass to assimilate Vr data (VrPass)

Problems with logarithmic transformation in CVlogq

- Due to the nonlinear logarithmic transformation between *q* and log(*q*), the effective spatial correlations of background error in *q* space is not Gaussian-like.

ρ_{ij} : log(q) background error correlation in q space

$\hat{\rho}_{ij}$: log(q) background error correlation in log(q) space (Gaussian like)

$$\rho_{ij} = \frac{[1+\sum_{k=2}^{\infty}(\ln 10)^{k-1}\delta\hat{q}_i^{\,k-1}(k!)^{-1}]}{[1+\sum_{k=2}^{\infty}(\ln 10)^{k-1}\delta\hat{q}_j^{\,k-1}(k!)^{-1}]} \cdot \frac{q_{b_i}}{q_{b_j}} \frac{\delta q_i}{\delta q_j} \hat{\rho}_{ij}$$

---

1. When background is homogeneous , and relatively larger than analysis increment

$$\rho_{ij} \approx \hat{\rho}_{ij}$$

2. When background is homogeneous , and relatively smaller than analysis increment

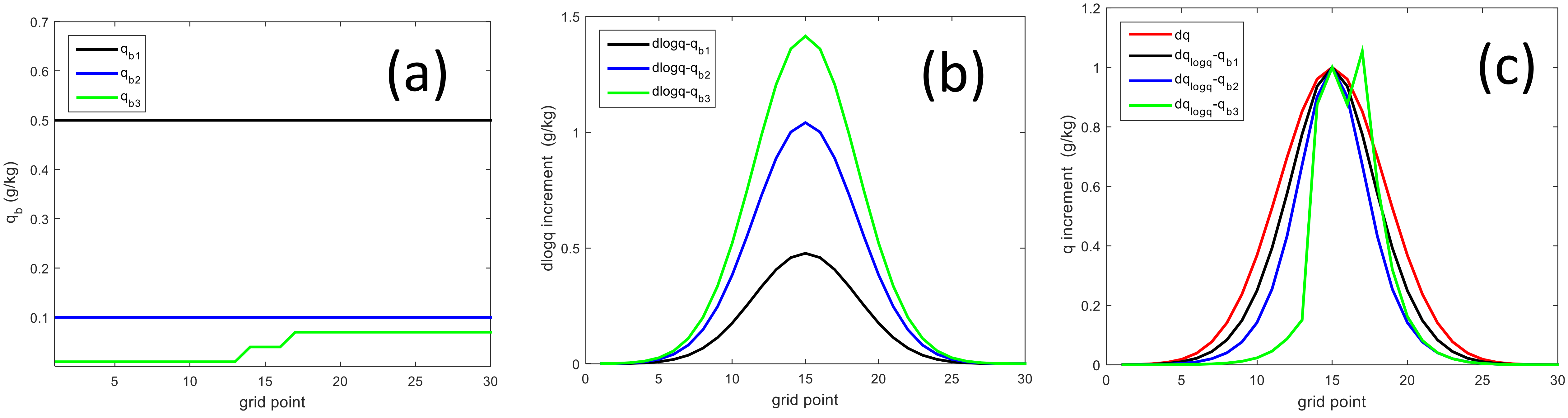
$$\rho_{ij} \leq \hat{\rho}_{ij}$$

3. When background is inhomogeneous , and relatively smaller than analysis increment

$$\rho_{ij} \approx \frac{q_{b_i}}{q_{b_j}} \cdot \hat{\rho}_{ij} = \begin{cases} > \hat{\rho}_{ij} & q_{b_i} > q_{b_j} \\ < \hat{\rho}_{ij} & q_{b_i} < q_{b_j} \end{cases}$$

Single observation test

- Assumptions:
  - Gaussian correlation functions with the same decorrelation scales are used for CVq and CVlogq
  - Same innovations (O-B)
  - Three different background values



Background | **Log(*q*) increment** | ***q* increment**

- When background is **homogeneous** , and relatively **larger** than analysis increment (**black line** in Fig. a), CVlogq analysis (**black line** in Fig. c) is closed to CVq analysis (**red line** in Fig. c)
- When background is **homogeneous** , and relatively **smaller** than analysis increment (**blue line** in Fig. a), the spread of analysis increment using CVlogq becomes narrow (**blue line** in Fig. c)
- When background is **inhomogeneous** , and relatively **smaller** than analysis increment (**green line** in Fig. a), the spread of analysis increment using CVlogq becomes non-Guassian (**green line** in Fig. c).

Special Treatment for CVlogq

- A lower limit is added to background when converting δq back to δq (X_bLim)

$$\hat{q} = \log(q)$$

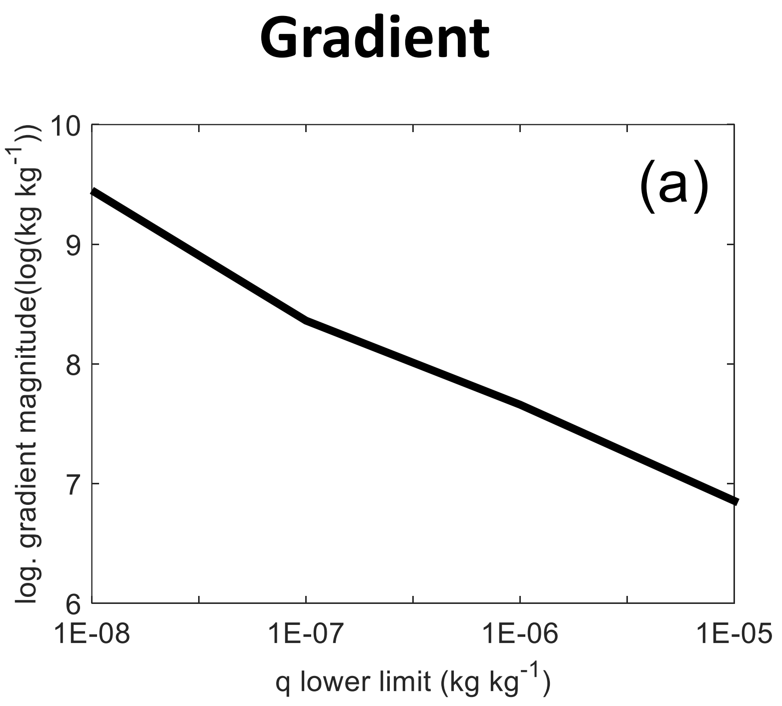
$$\delta q = 10^{\delta q + \log(\max(q_b, \varepsilon_b))} - \max(q_b, \varepsilon_b)$$

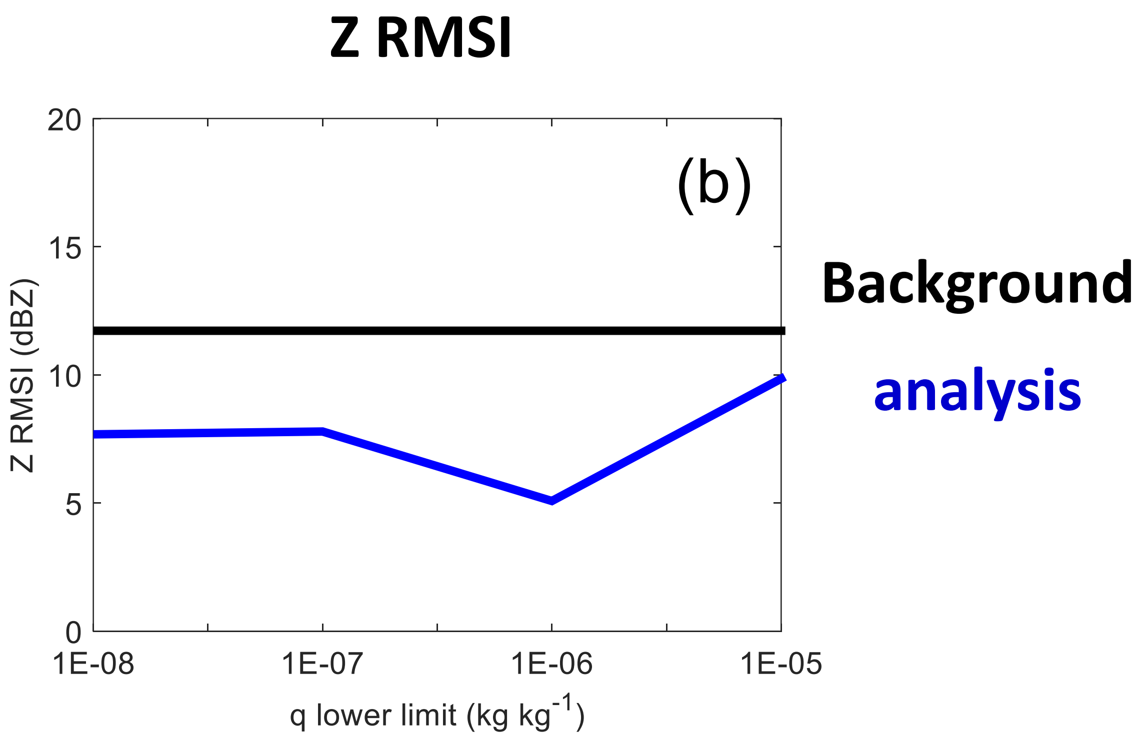
Examination of the treatments through OSSEs

- Tested with simulated data from a supercell storm of 20 May 1977 near Del City, Oklahoma
- ARPS-3DVar system and ARPS model
- 2 km horizontal and 0.5 km vertical grid spacings

	CVq				**CVlogq**	
treatment	qLim	Z_eLim	qLim+VrPass	ZeLim+VrPass	No Treatment	X_bLim
Exp. name	CVq-qLim	CVq-Z_eLim	CVq-qLim-VrP	CVq-ZeLim-VrP	CVlogq-NT	CVlogq-X_bLim

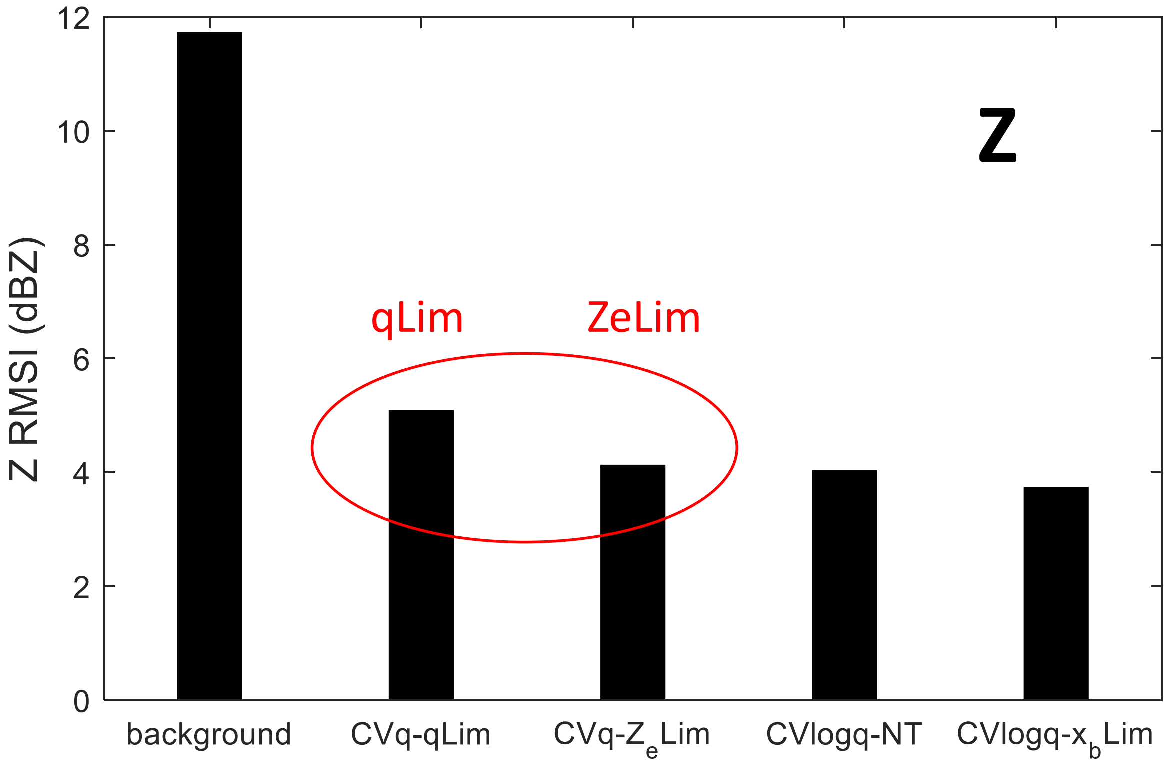
Optimal threshold for qLim treatment

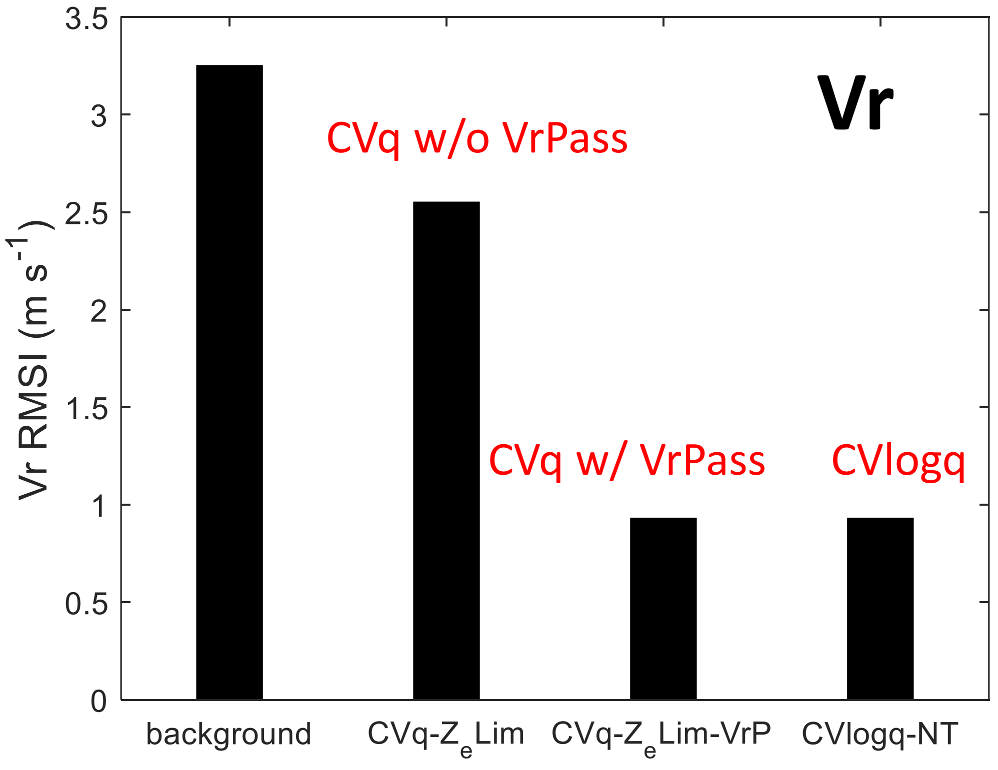




- The magnitude of the gradient significantly decreases when increasing the qLim threshold.
- 10^{-6} kg kg^{-1} is the optimal threshold of hydrometeor mixing ratio in this study.

RMSI for Z and Vr





- Z_eLim is better than qLim in terms of a smaller RMSI
- Vr data is difficult to assimilate when Z and Vr data are assimilated together using CVq.
- the Vr RMSIs of CVq with VrPass treatment is similar to CVlogq

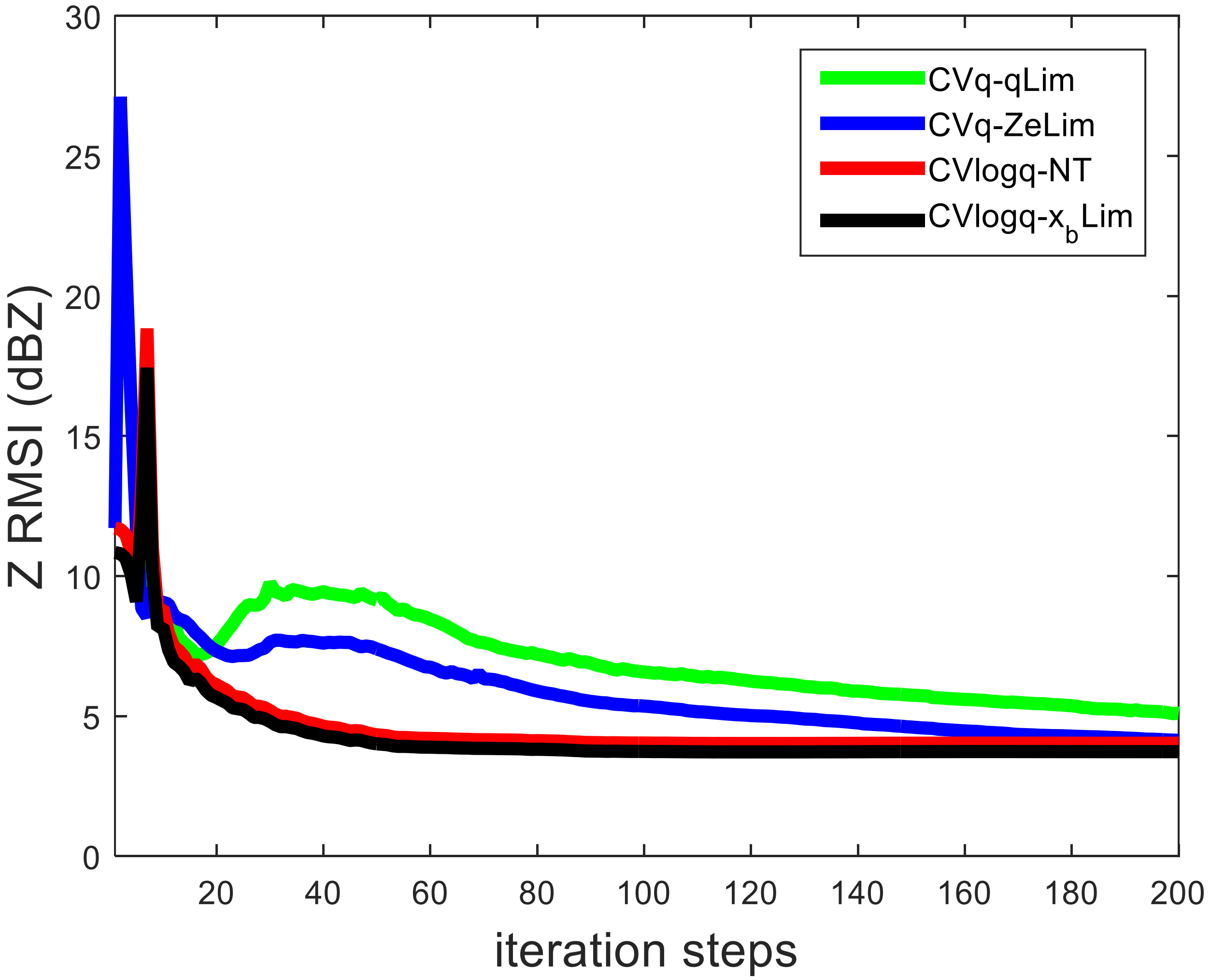
Convergence speeds of using CVq and CVlogq

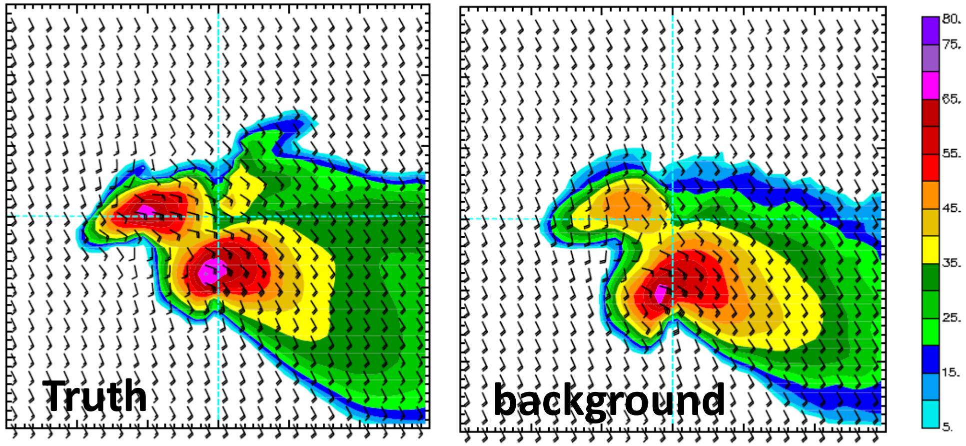
CVlogq converges much faster than CVq.

Reason:

the gradient range :

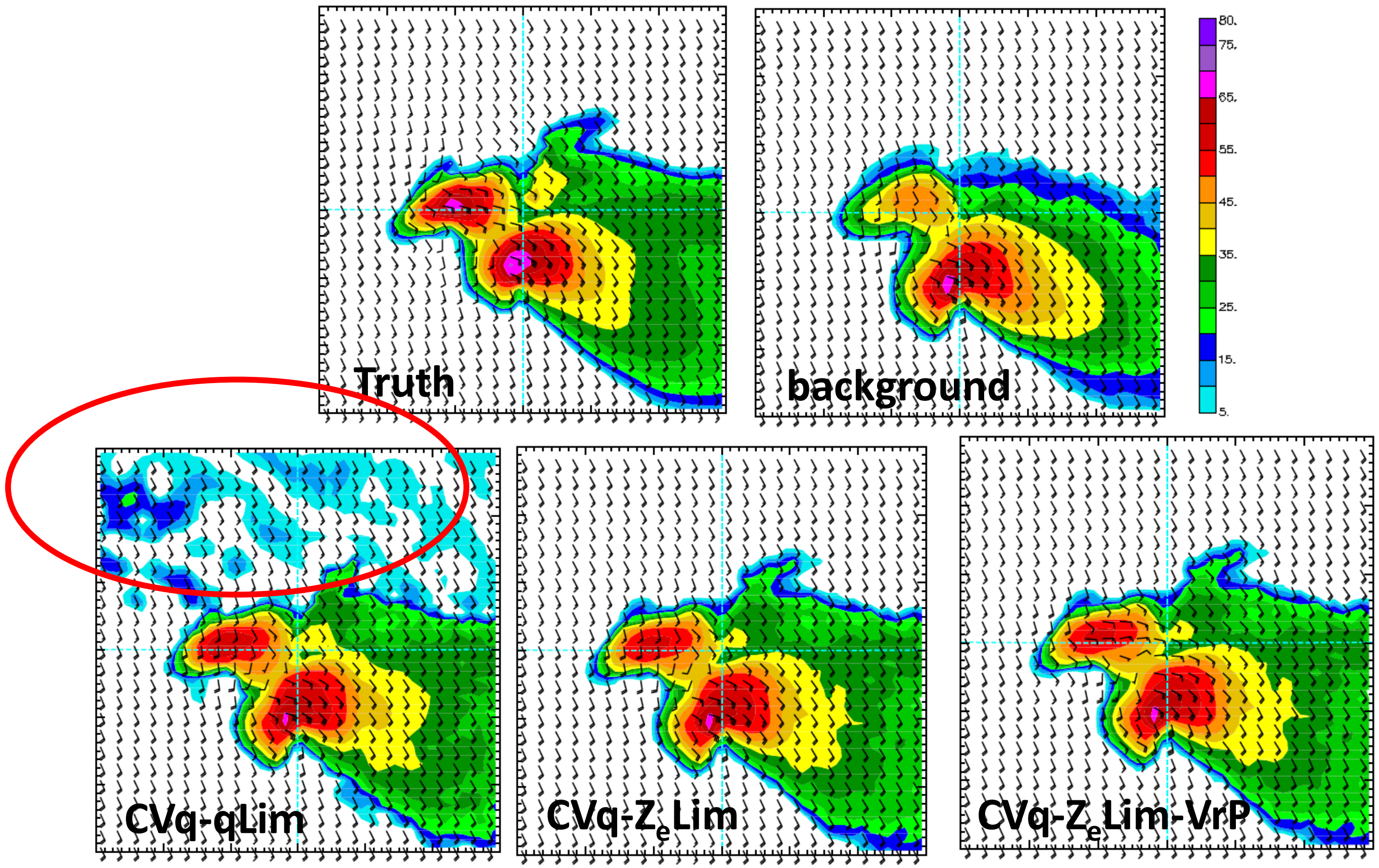
CVq: -10^8 to 10^8 CVlogq: -10^2 to 10^2





Truth
background
80.
75.
65.
55.
45.
35.
25.
15.
5.

- Applying qLim can introduce spurious Z analyses.

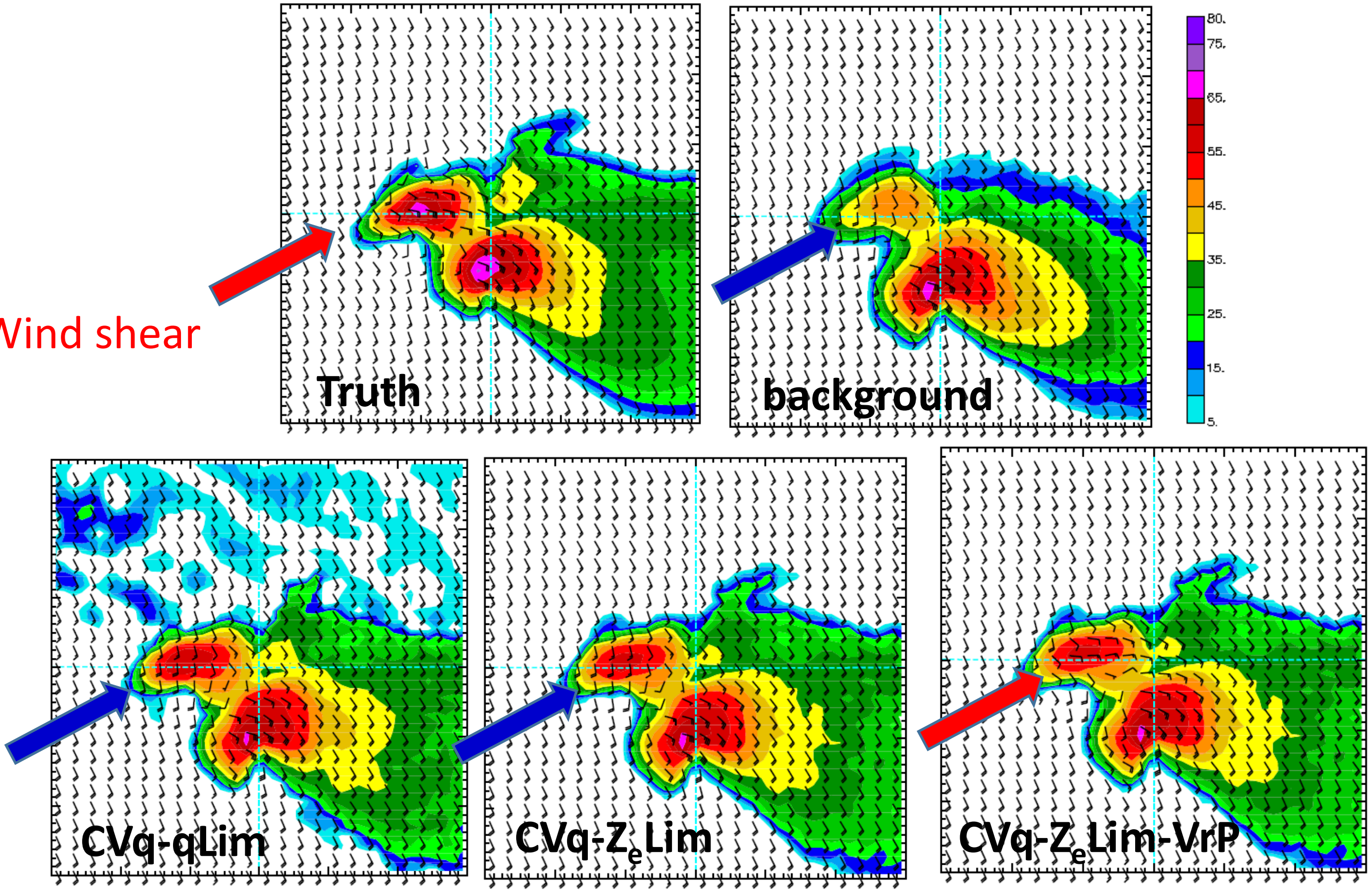


- Applying qLim can introduce spurious Z analyses.

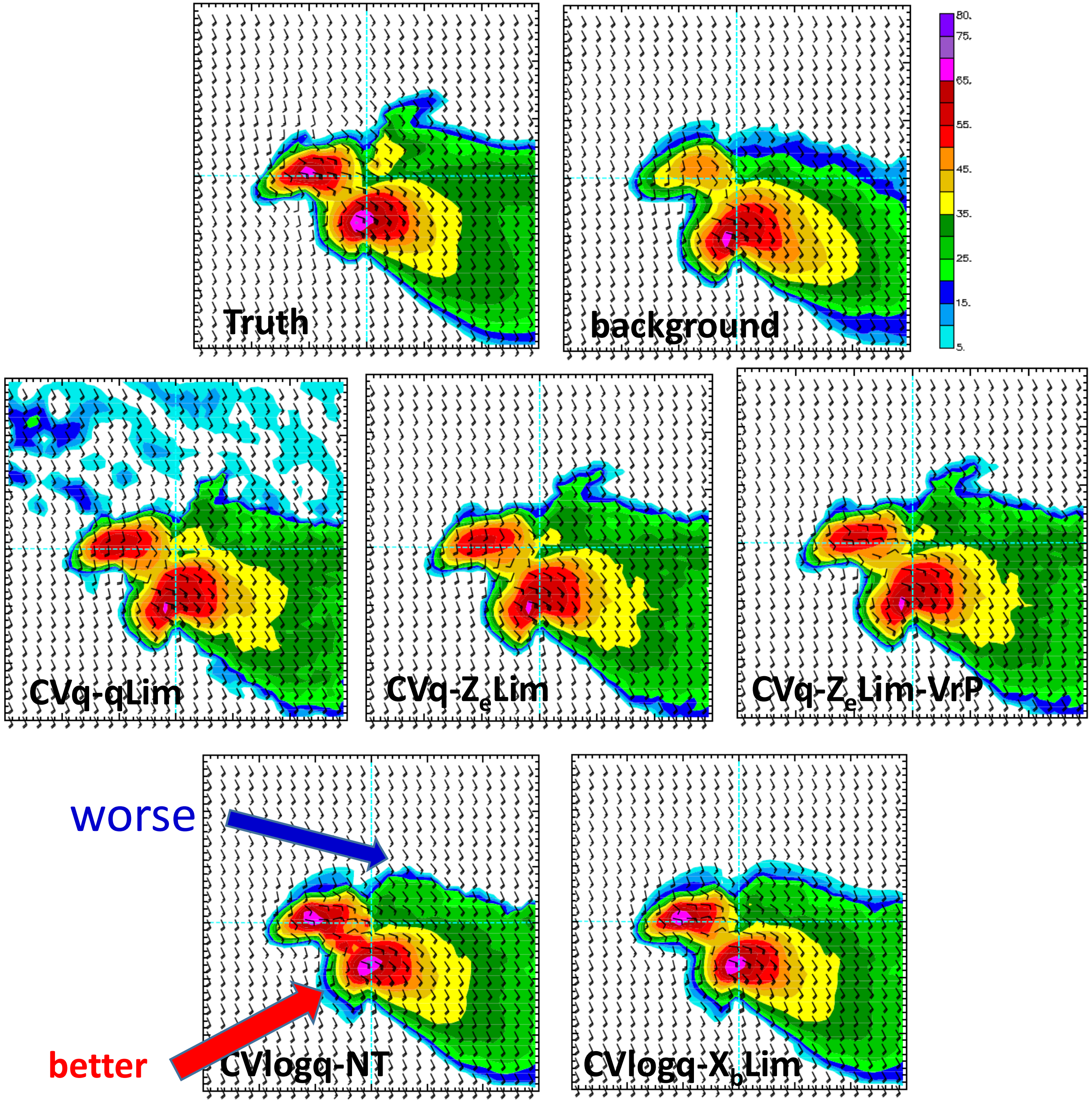
- Using different pass for radial velocity DA, the analysis is greatly improved.

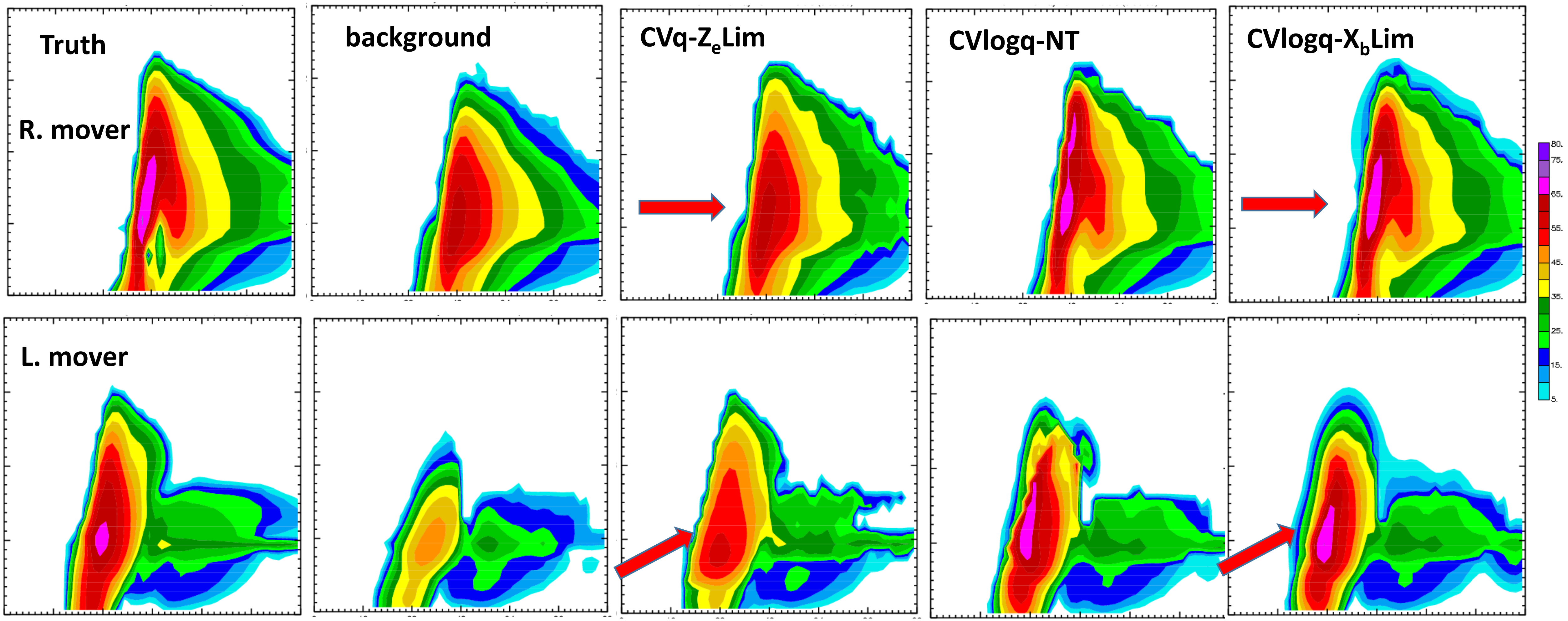
Low-level Wind shear

Missing
Low-level wind shear

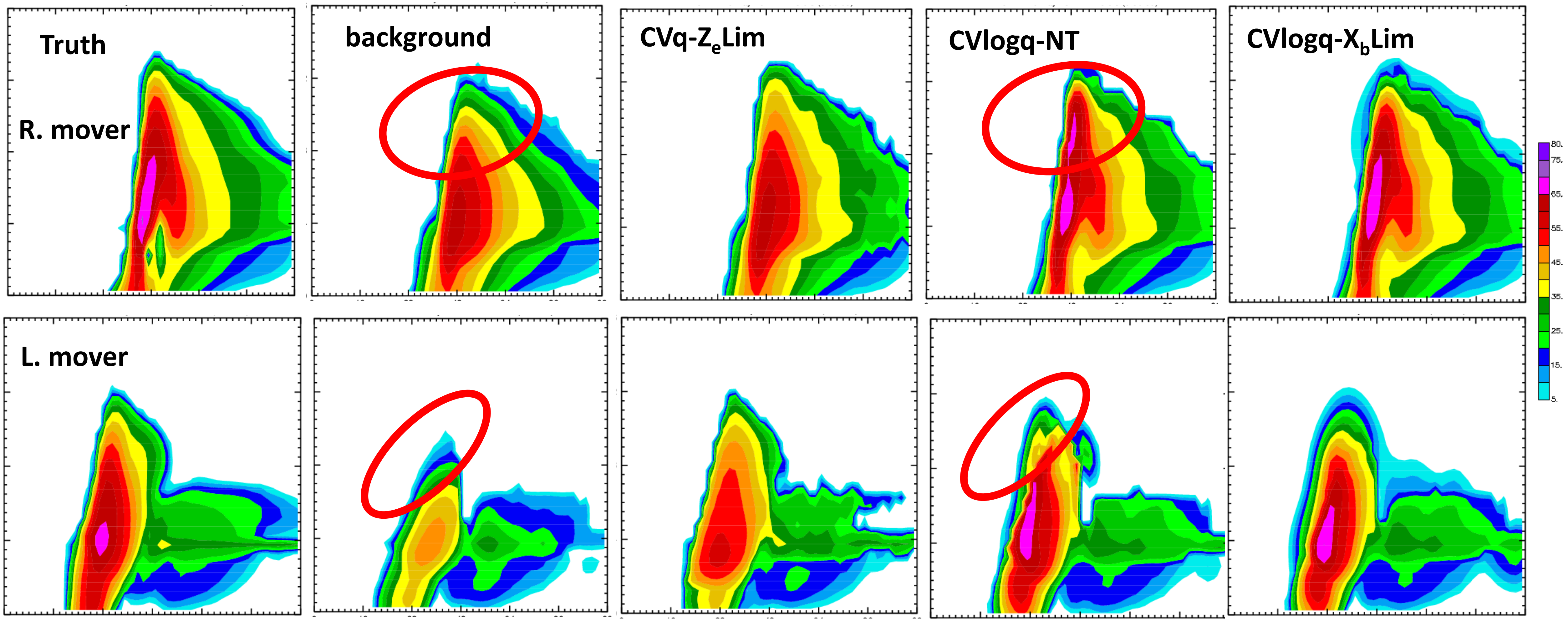


- Applying qLim can introduce spurious Z analyses.
- Using different pass for radial velocity DA, the analysis is greatly improved.
- CVlogq outperforms CVq for analysis of the reflectivity core, but underperforms for analysis of small reflectivity



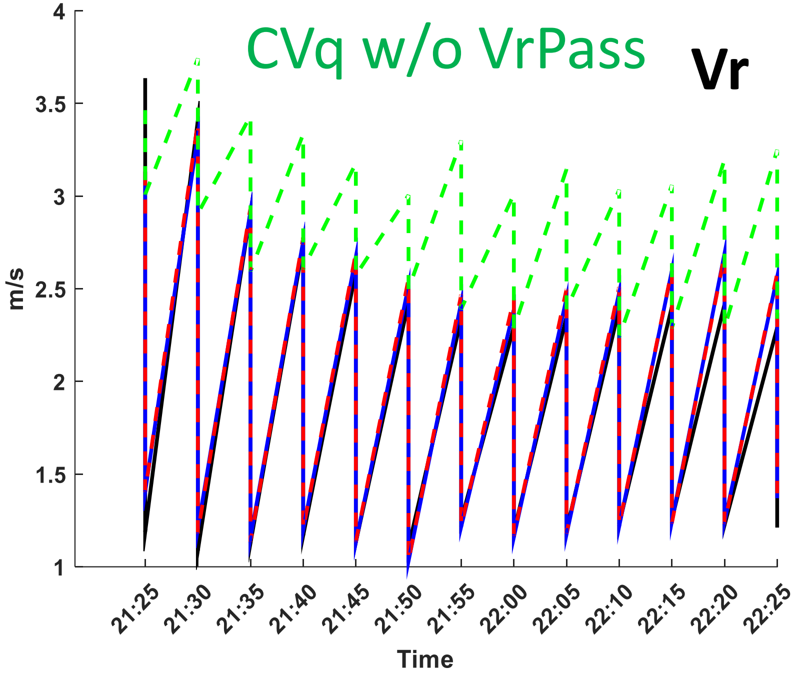


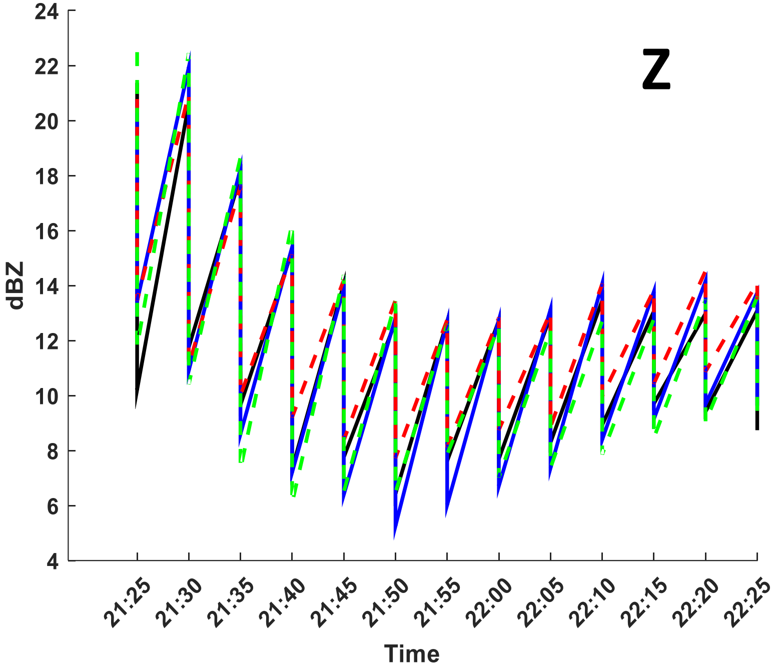
- CVlogq performs betters for the analysis of the high reflectivity cores



- CVlogq performs betters for the analysis of the high reflectivity cores.
- X_bLim treatment can reduce the improper spreading of the analysis increment for CVlogq.

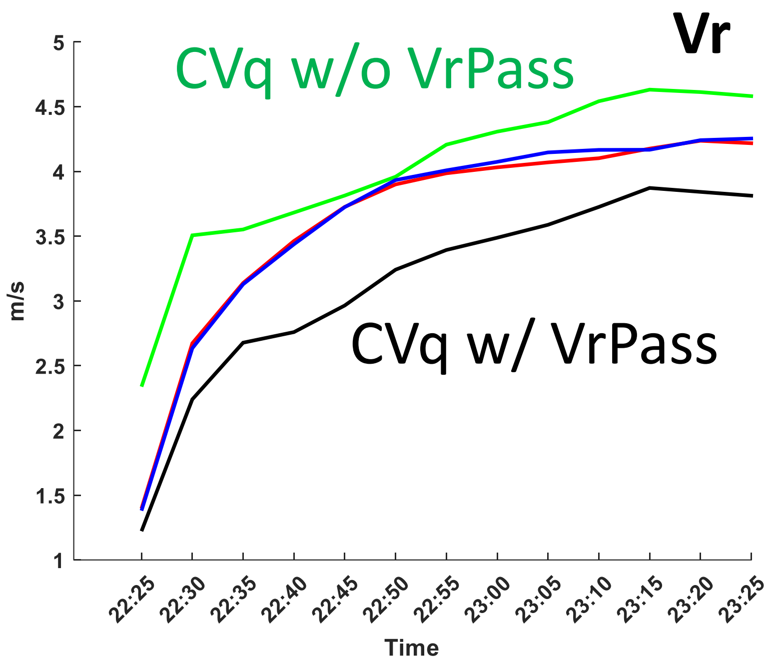
RMSI of one hour cycle

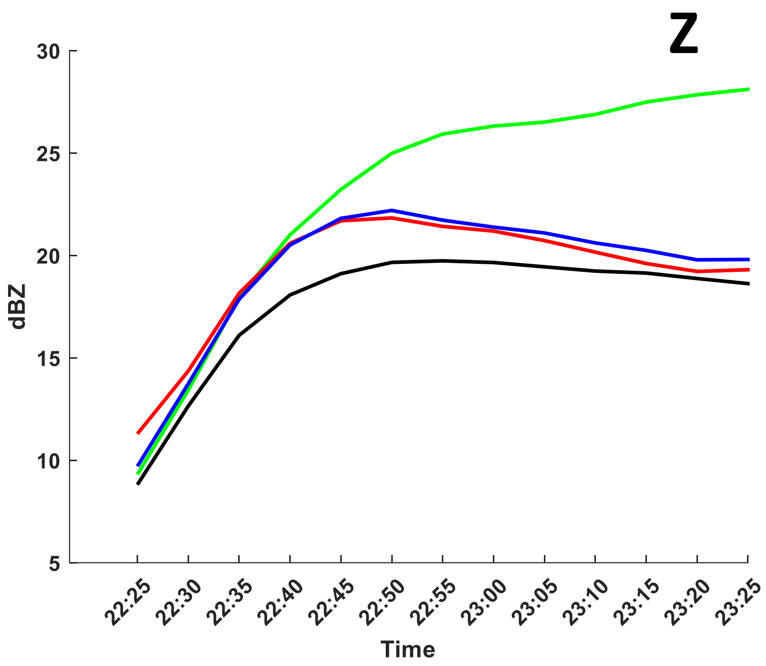




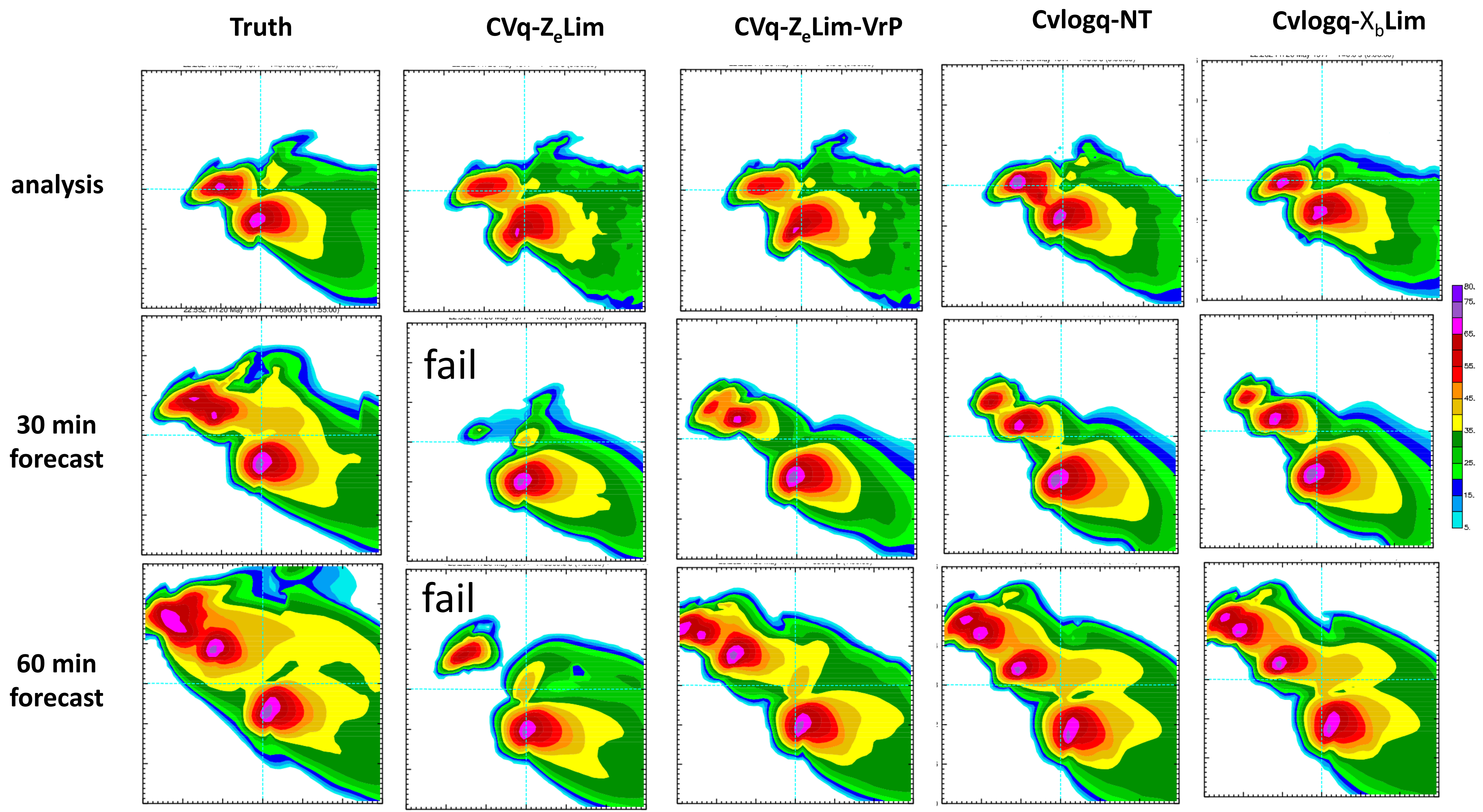
- Green: CVq-Z_eLim Black: CVq-Z_eLim-VrP Red: Cvlogq-NT Blue: Cvlogq-X_bLim

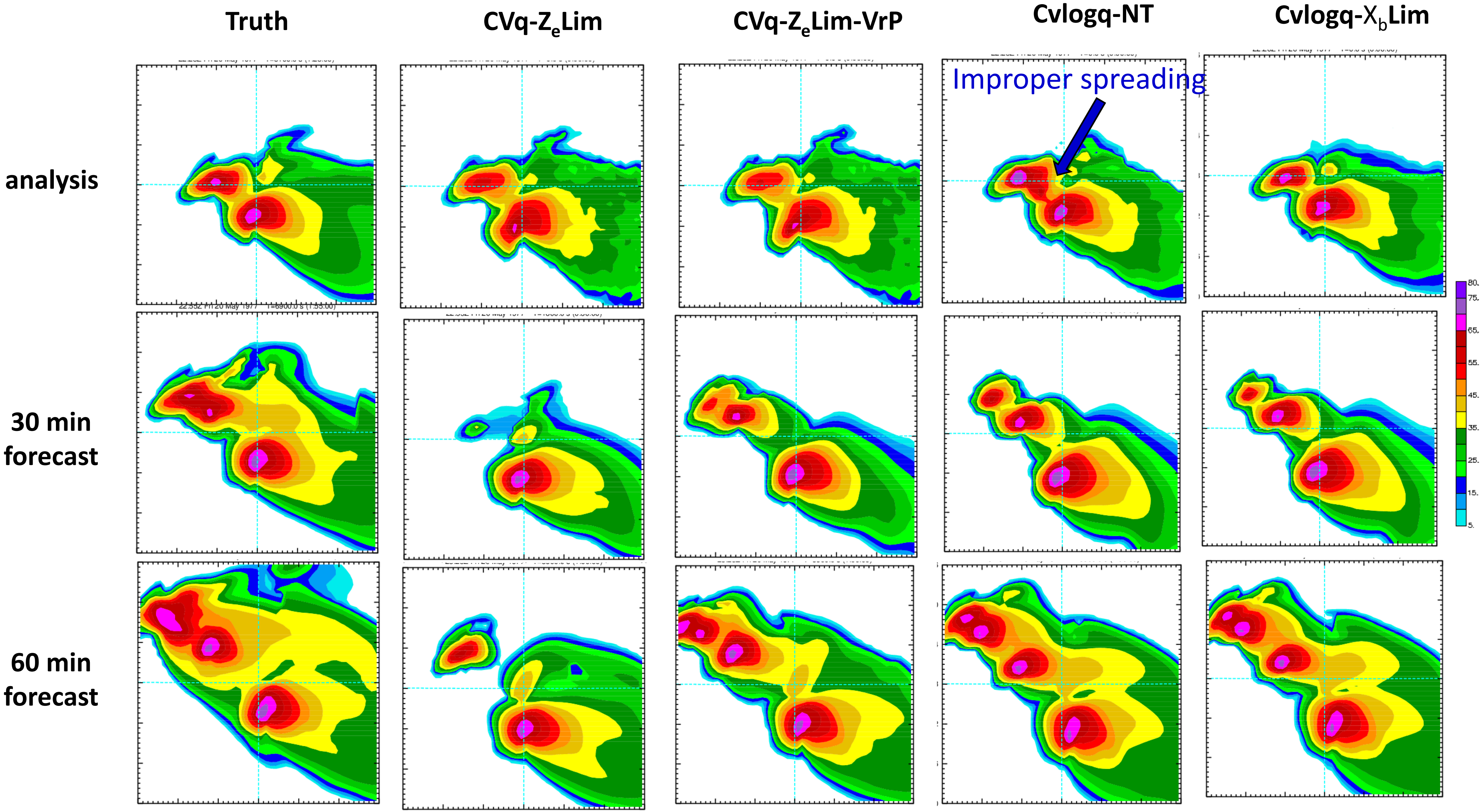
RMSEs for one hour forecast



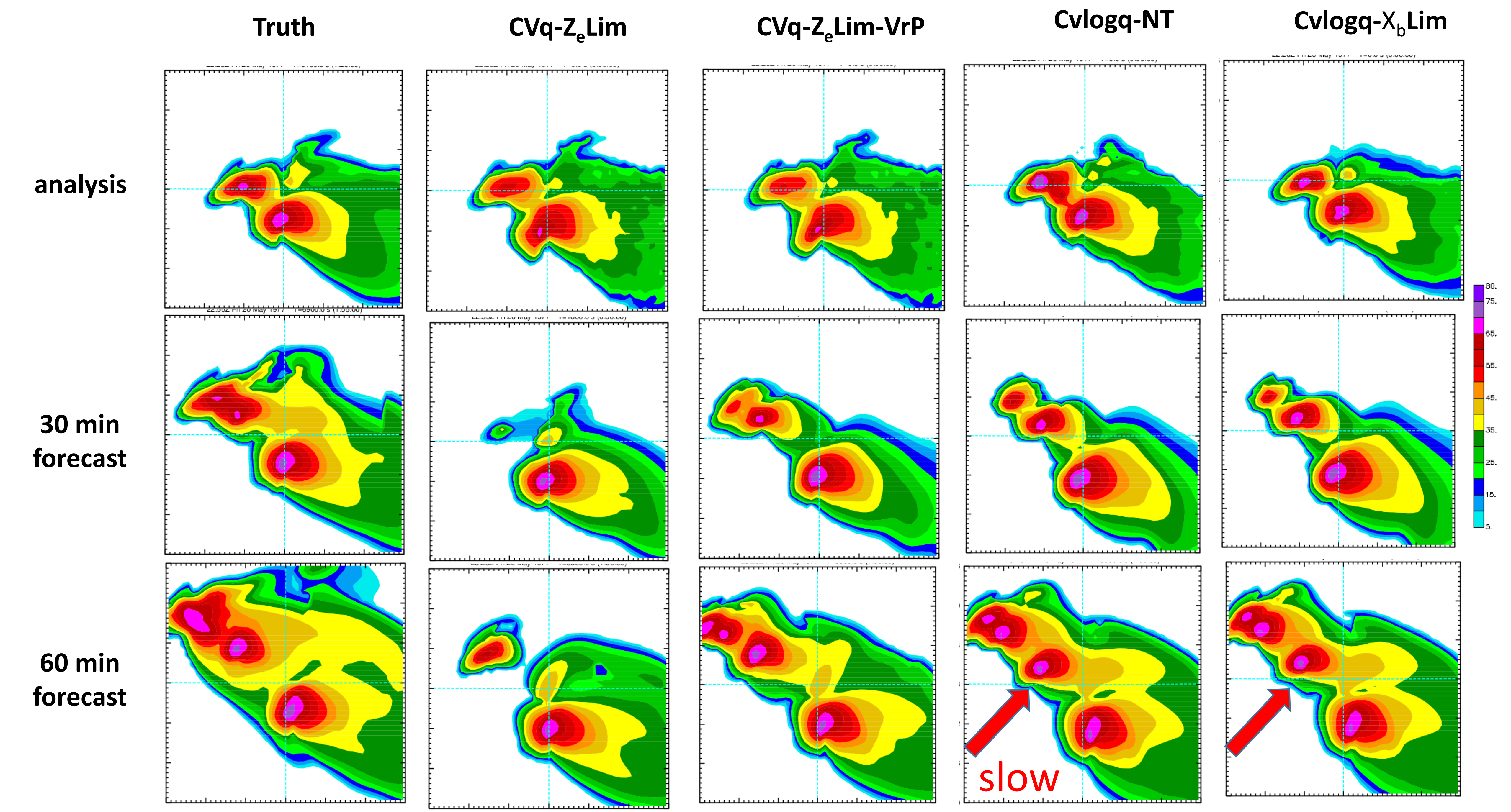


Green: CVq-Z_eLim Black: CVq-Z_eLim-VrP Red: Cvlogq-NT Blue: Cvlogq-X_bLim


Truth
CVq-Z_eLim
CVq-Z_eLim-VrP
Cvlogq-NT
Cvlogq-X_bLim
analysis
30 min forecast
60 min forecast
fail
fail
80.
75.
65.
55.
45.
35.
25.
15.
5.



Truth
CVq-Z_eLim
CVq-Z_eLim-VrP
Cvlogq-NT
Cvlogq-X_bLim
analysis
30 min forecast
60 min forecast
Improper spreading
80.
75.
65.
55.
45.
35.
25.
15.
5.



Truth
CVq-Z_eLim
CVq-Z_eLim-VrP
Cvlogq-NT
Cvlogq-X_bLim
analysis
30 min forecast
60 min forecast
slow
80.
75.
65.
55.
45.
35.
25.
15.
5.

Summary

	Extremely Large gradient	Lacking adjustment for Vr analysis for Vr+Z DA	Improper spreading of Z analysis increment	Lacking adjustment for Z analysis
CVq	Yes, need qLim/Z_eLim, Z_eLim better	Yes, need VrPass	No	Yes, for large background value (i.e. reflectivity core)
CVlogq	No	No	Yes. Need X_bLim	Yes, for small background value

Ongoing research

- Apply the proposed treatments to an EnVar DA system.