EFSR: Ensemble Forecast Sensitivity to Observation Error Covariance

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1. Motivation

- Every standard DA textbook starts by assuming that
  $$\mathbf{x}^b \sim \mathcal{N}(\mathbf{x}^{true}, \mathbf{B}), \qquad \mathbf{y}^o \sim \mathcal{N}(\mathbf{y}^{true}, \mathbf{R})$$
  as if **B** and **R** are known.
- However, **B** and **R** are unknown external parameters that, in practice, have to be somehow estimated, often subject to *empirical/subjective tuning*. In this study we focus on how to estimate **R**.
- Standard methods to estimate **R** rely on residual statistics combined with some ad-hoc assumptions:
  - Hollingsworth and Lönnberg (1986): assume diagonality of **R**.
  - Desroziers et al. (2005): assume optimality of the currently-tested DA system; iteratively correct if diagnostics disagrees with the currently-tested system.
- Alternative approach by Daescu (2008):
  - Diagnose how a small change to **R** would increase/decrease a quadratic forecast error aspect using the adjoint sensitivity technique
  - Then use the diagnostics as a guide to tune **R** so that forecast error would be reduced.
  - Powerful diagnostics, but requires the tangent linearization/adjoint of the forecast model
- **Objective of this study:**
  - **to formulate an ensemble-based equivalent of Daescu's adjoint-based R-sensitivity diagnostics**
  - **to assess effectiveness of its application to R-tuning**

2. Formulation

- Define the forecast error as $\mathbf{e}_{t|0} = \mathbf{x}^f_{t|0} - \mathbf{x}^v_t$ and its quadratic aspect $e^f_{t|0} = \mathbf{e}^T_{t|0}\mathbf{C}\mathbf{e}_{t|0}$

  where $\mathbf{x}^f_{t|0}$ is the forecast valid at time t initialized at time $t = 0$, $\mathbf{x}^v_t$ is the verifying state at time t, and $\mathbf{C}$ is a square positive-definite matrix that defines the error norm
- **R**-sensitivity derivation by Daescu (2008) requires the adjoint of the forecast model ($\mathbf{M}_{t|0}$) and the data assimilation (**K**).

$$\frac{\partial e^f_{t|0}}{\partial R_{i,j}} = -(\mathbf{R}^{-1}\delta\mathbf{y}^{oa})_j \left(\frac{\partial e^f_{t|0}}{\partial \mathbf{y}^o}\right)_i$$

with $$\frac{\partial e^f_{t|0}}{\partial \mathbf{y}^o} = 2\mathbf{K}^T\mathbf{M}^T_{t|0}\mathbf{C}\mathbf{e}_{t|0}$$

Daescu (2008)

- Within an EnKF, adjoint evaluation can be alleviated following the derivation of EFSO by Kalnay et al. (2012).

Hotta et al. (2017)

$$\frac{\partial e^f_{t|0}}{\partial R_{i,j}} = -(\mathbf{R}^{-1}\delta\mathbf{y}^{oa})_j(2\mathbf{K}^T\mathbf{M}^T_{t|0}\mathbf{C}\mathbf{e}_{t|0})_i$$
$$= -(\mathbf{R}^{-1}\delta\mathbf{y}^{oa})_j \frac{2}{K-1}[\mathbf{R}^{-1}\mathbf{Y}^a(\mathbf{M}_{t|0}\mathbf{X}^a)^T\mathbf{C}\mathbf{e}_{t|0}]_i$$
$$\approx -(\mathbf{R}^{-1}\delta\mathbf{y}^{oa})_j \frac{2}{K-1}(\mathbf{R}^{-1}\mathbf{Y}^a\mathbf{X}^{fT}_{t|0}\mathbf{C}\mathbf{e}_{t|0})_i,$$

$$\because \mathbf{K} = \mathbf{A}\mathbf{H}^T\mathbf{R}^{-1} \approx \frac{1}{K-1}(\mathbf{X}^a\mathbf{X}^{aT})\mathbf{H}^T\mathbf{R}^{-1} = \frac{1}{K-1}\mathbf{X}^a\mathbf{Y}^{aT}\mathbf{R}^{-1}$$

Kalnay et al. (2012)

3. Idealized experiments with Lorenz '96 model

Experimental Set-up

Expt	True observation error variance	Prescribed obs. err. var.
SPIKE	$\sigma_j^{o,true^2} = \begin{cases} 0.8^2, & j = 11 \\ 0.2^2, & j \neq 11 \end{cases}$	$\sigma_j^{o^2} = 0.2^2$ everywhere
STAGGERED	$\sigma_j^{o,true^2} = \begin{cases} 0.1^2, & j\text{: odd} \\ 0.3^2, & j\text{: even} \end{cases}$	$\sigma_j^{o^2} = 0.2^2$ everywhere

Results

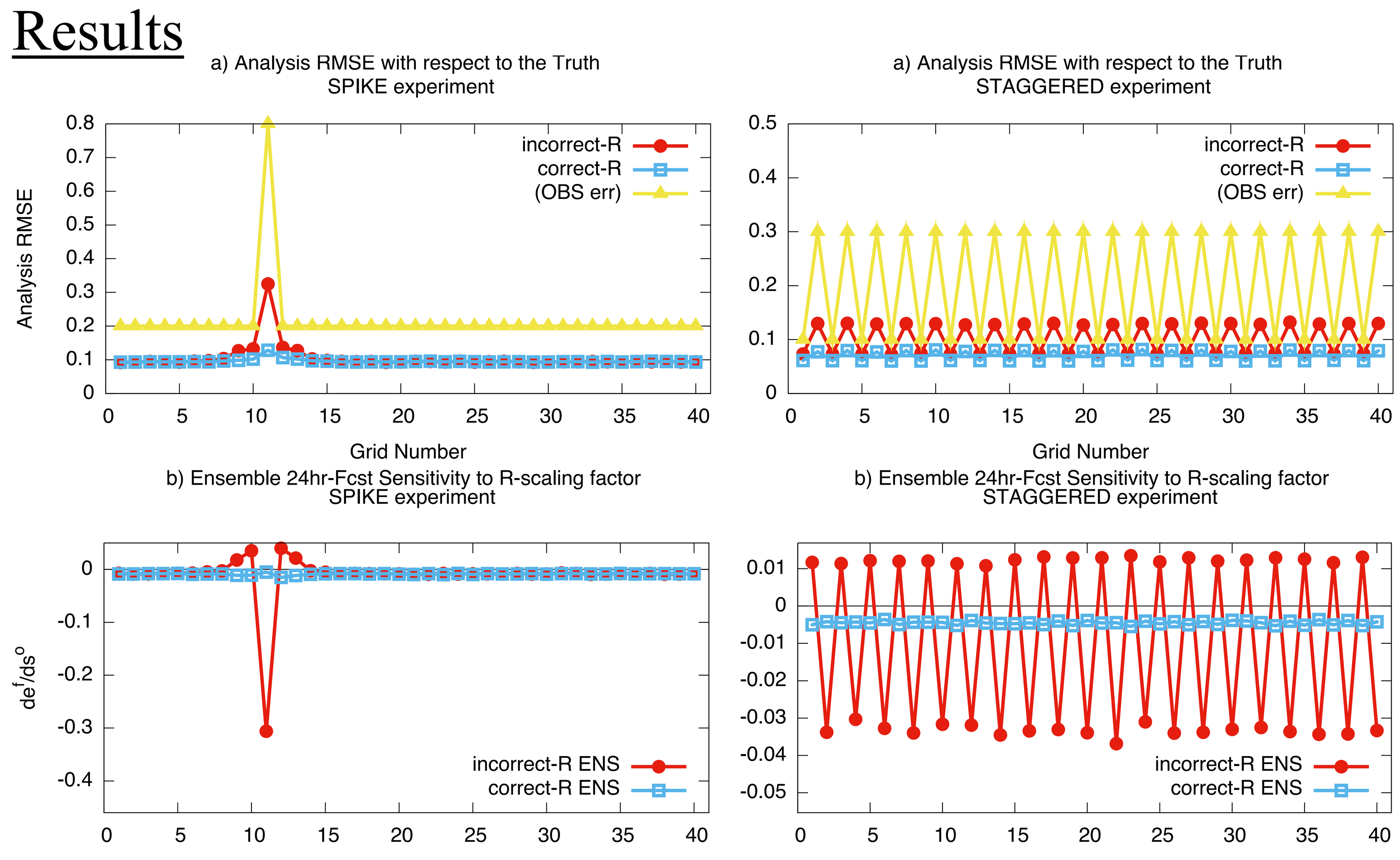


FIG. 1. (a) Analysis errors verified against the truth for the SPIKE experiment displayed as a function of grid number. The filled circles and open squares show the analysis errors, respectively, for the incorrect-**R** and correct-**R** runs. As a reference, observation errors verified against the truth are also shown by the triangles. (b) Ensemble-based 24-h forecast sensitivity to the **R**-scaling factors for the incorrect-**R** (filled circles) and correct-**R** (open squares) runs of the SPIKE experiment.

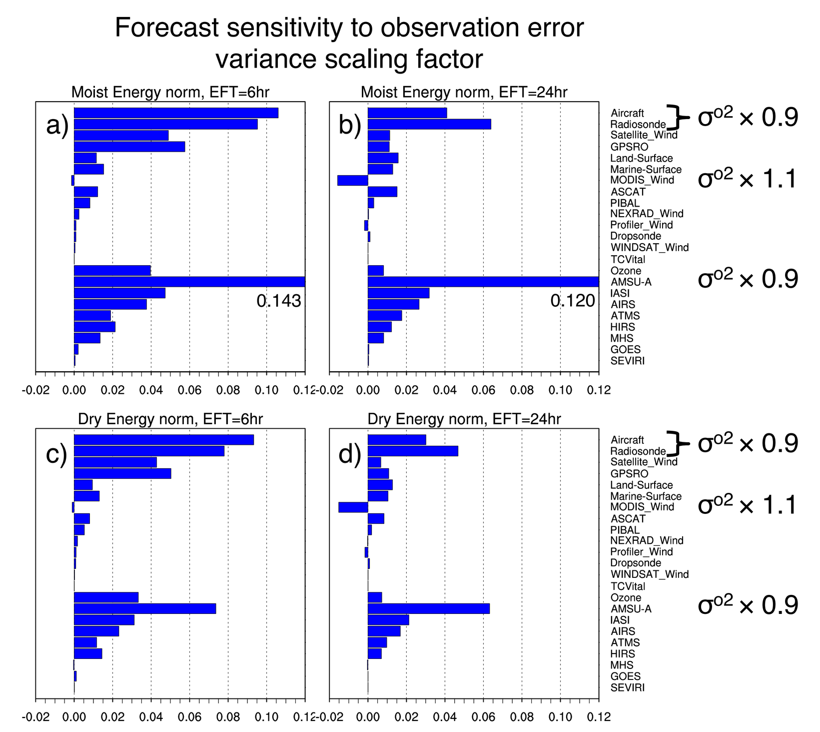
FIG. 2. As in Fig. 1, but for the STAGGERED experiment.

EFSR successfully diagnoses whether σ^{o2} should be increased or decreased.

4. Experiments with an quasi-operational system

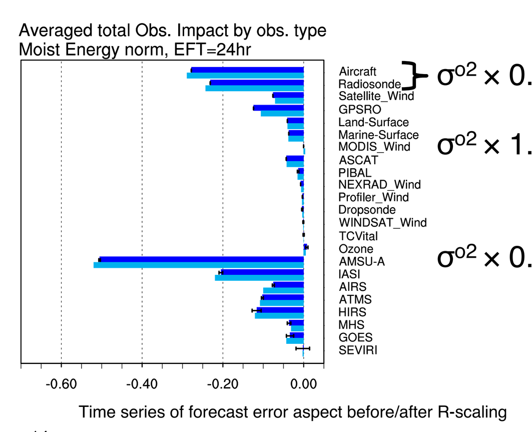
EFSR diagnostics for the NCEP's GFS hybrid GSI coupled with LETKF

- Positive **R**-sensitivity for most observation types except for MODIS wind.
- Pos/neg sensitivity implies that **R** should be reduced/increased.



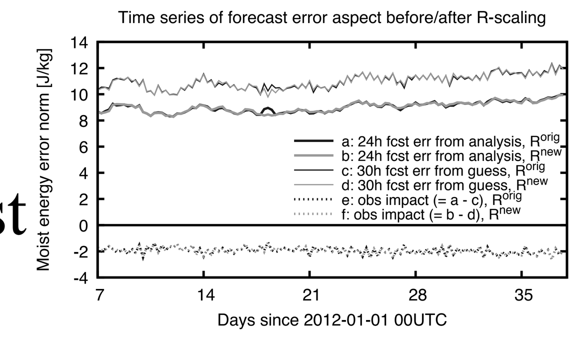
R-sensitivity experiment:

- **R** for three obstypes (Aircraft, Radiosonde and AMSU-A) with large positive sensitivity reduced by x0.9, **R** for MODIS wind scaled by 1.1.



Results:

- EFSO for the tuned obstypes enhanced
- but no statistically significant forecast error reduction.



5. Summary

- Ensemble-based R-sensitivity successfully formulated
- Worked very well for idealized experiments
- More work required to improve operational system
- Details published in our *MWR* paper available online at https://doi.org/10.1175/MWR-D-17-0122.1 (open access)

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