Treating sample covariances for use in strongly-coupled atmosphere-ocean data assimilation

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Aims

- Strongly-coupled variational data assimilation for coupled systems requires the specification of cross-domain forecast error covariances.
- Ensembles can be used to estimate these, but sample covariances are typically rank deficient and/or ill-conditioned.

➢ How can we obtain a well-conditioned matrix that retains important cross-covariance information?

Condition number

We recall that for a symmetric positive definite matrix **S**, the condition number is given by

$$\kappa(\mathbf{S}) = \lambda_{\mathsf{max}}(\mathbf{S})/\lambda_{\mathsf{min}}(\mathbf{S})$$

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Matrix modification methods

1. Matrix reconditioning
   - Specify a required condition number κ_{tol} and increment all eigenvalues by a fixed amount λ_{inc} such that

$$\frac{\lambda_{\max} + \lambda_{\text{inc}}}{\lambda_{\min} + \lambda_{\text{inc}}} = \kappa_{\text{tol}}$$

   - Note that reconditioning the covariance matrix in this way is not the same as reconditioning the correlation matrix.

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Matrix modification methods

2. Localization
   - Form Schur product of localization matrix and ensemble covariance matrix.
   - For coupled DA need to think carefully how to apply to multivariate blocks and sub-matrices

$$\mathbf{C} = \begin{pmatrix} \mathbf{C}_{\mathbf{AA}} & \mathbf{C}_{\mathbf{AO}} \\ \mathbf{C}_{\mathbf{AO}}^{T} & \mathbf{C}_{\mathbf{OO}} \end{pmatrix}$$

   - We apply separate localization to each sub-matrix.

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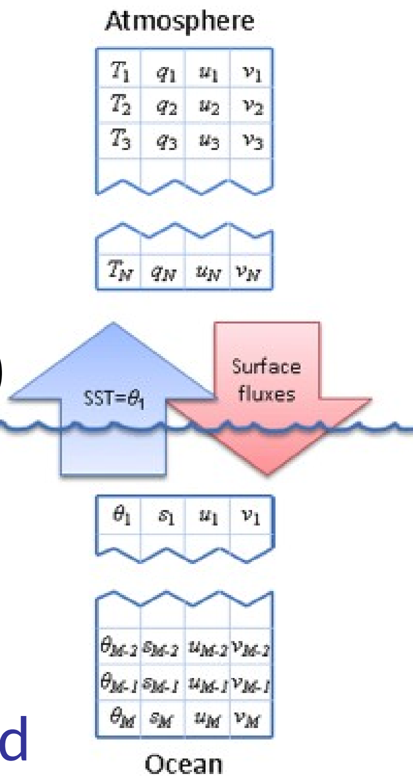
Idealised system

single-column, coupled atmosphere-ocean model

Atmosphere

simplified version of the ECMWF single column model
adiabatic component + vertical diffusion (no convection)
4 state variables on 60 model levels (surface to ~0.1hPa)
forced by large scale horizontal advection



Ocean

K-Profile Parameterisation (KPP) mixed-layer model based on the scheme of *Large et al*
4 state variables on 35 model levels (1-250m)
forced by short and long wave radiation at surface

coupled via SST and surface fluxes of heat, moisture & momentum

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Ensemble of 4D-Vars

Estimate background error covariance from a ensemble of perturbed strongly coupled 4D-Var analyses

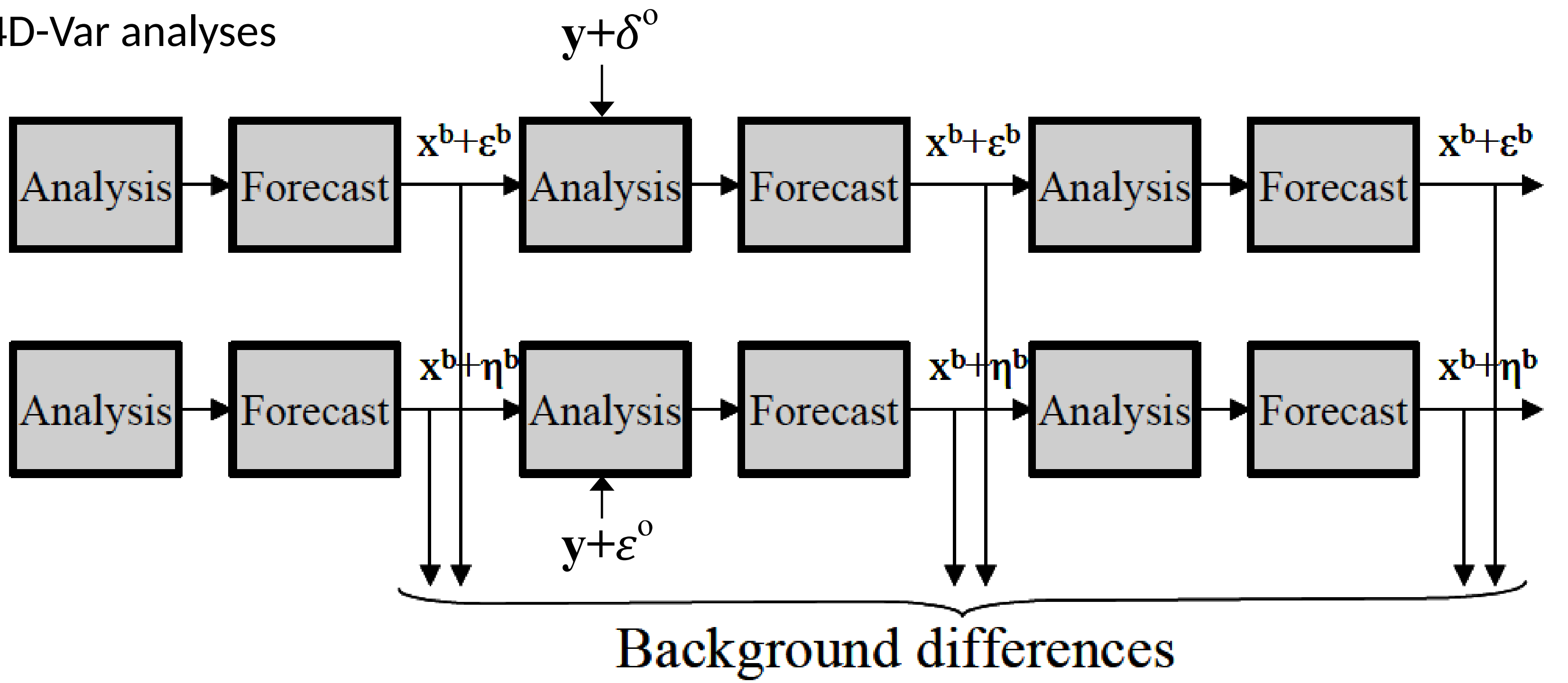

Figure 3: Schematic illustration of the analysis-ensemble method of generating fields of background difference.

adapted from Fisher 2003

Differences between pairs of background fields have same **correlation** structure as background error (but twice the variance)

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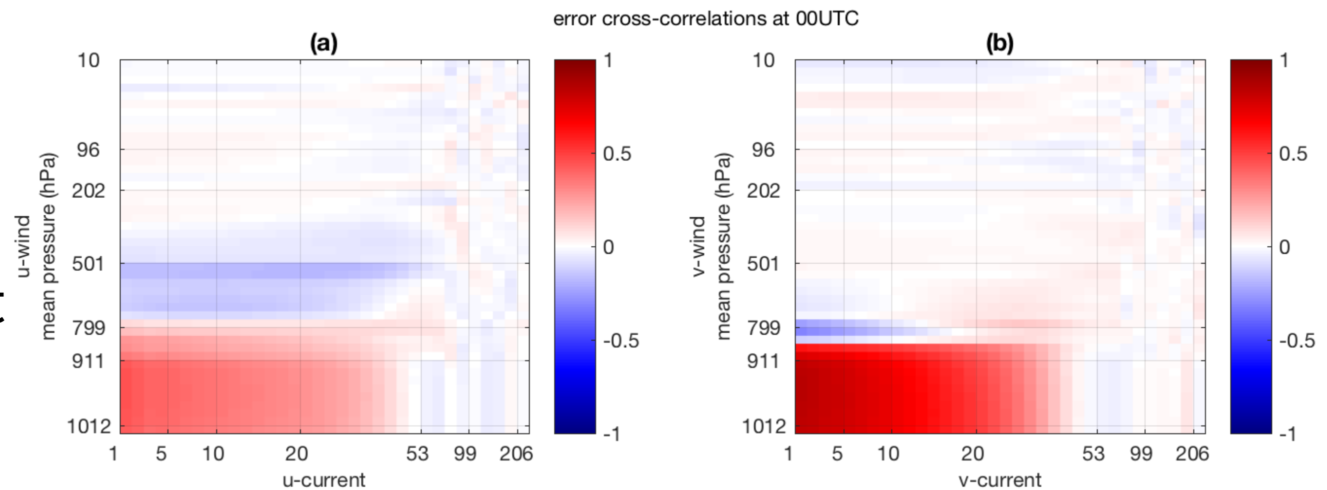
Example correlations, 500 members December 2013, NW Pacific

u-wind
&
u-current



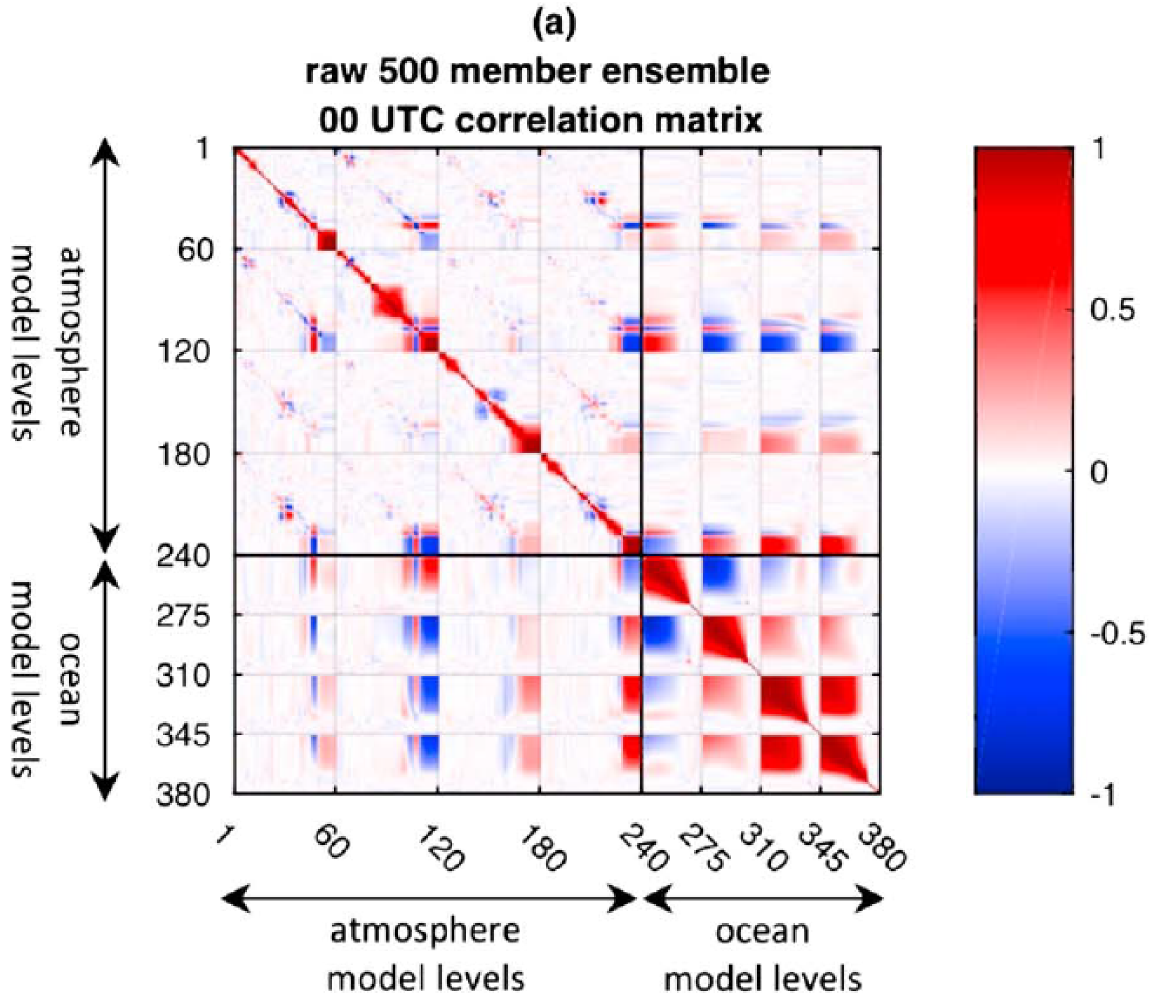
v-wind
&
v-current

Smith et al. (2017)

depth (m)

depth (m)

Full ensemble correlation matrix



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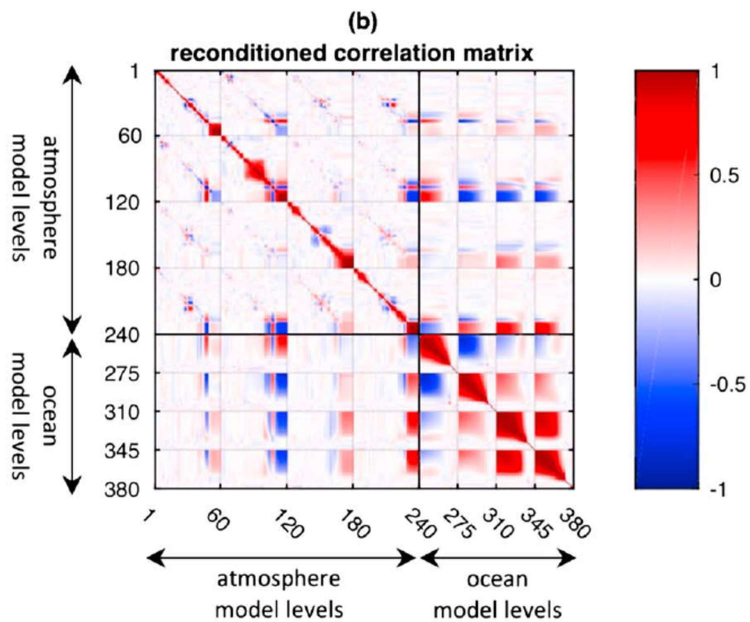
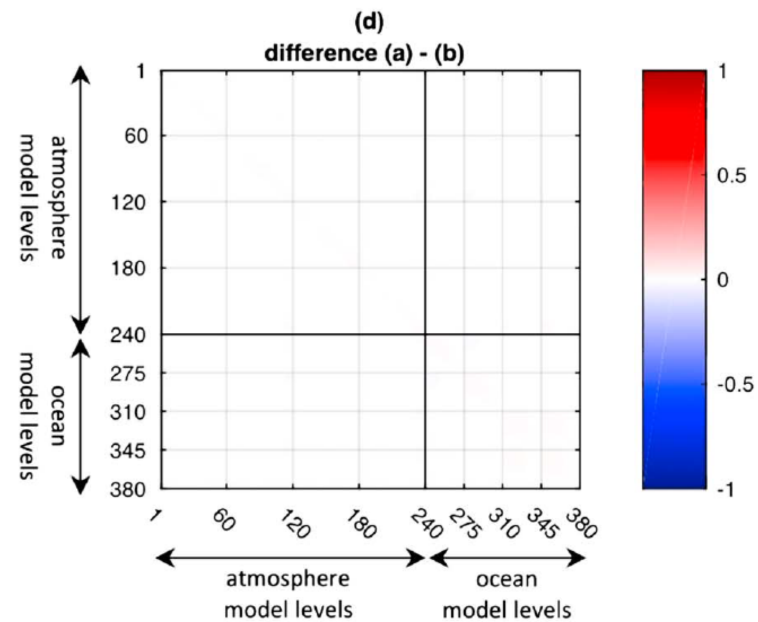
Reconditioned correlation matrix, $\kappa=10^4$



Reconditioned



Difference from original

Condition number reduced and correlation structure retained, but so is sampling noise.

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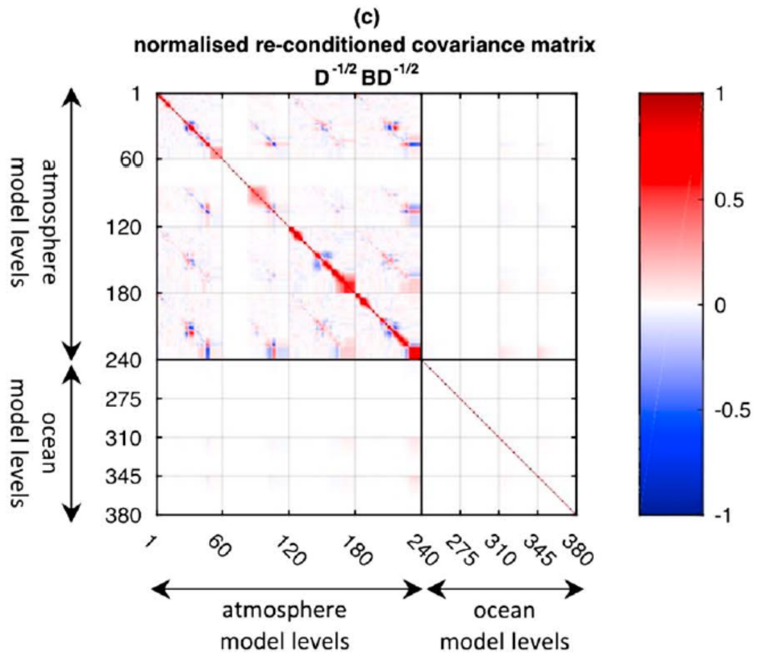
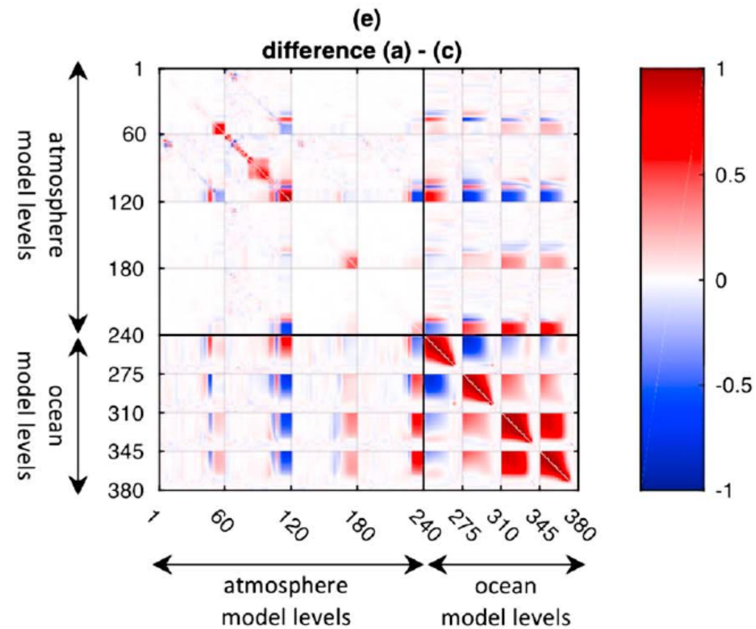
Reconditioned covariance matrix, κ=10^4



Reconditioned



Difference from original

Structure of ocean cross correlations and atmosphere-ocean cross correlations is lost.

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Localization

We define a scaled distance between an atmosphere and ocean point (similar to Frolov *et al., 2016).*

$$\hat{d}(z_a(i), z_o(j)) = \left(\frac{z_a(i)}{L_a} + \frac{z_o(j)}{L_o} \right)$$

The same localization length scale is used for each sub-matrix in the atmosphere and same in the ocean – Helps keep matrix positive definite.

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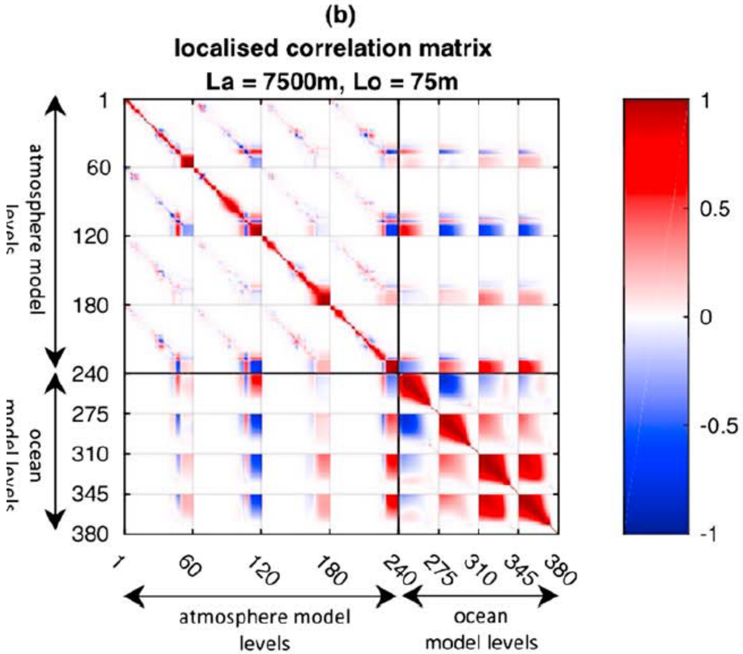
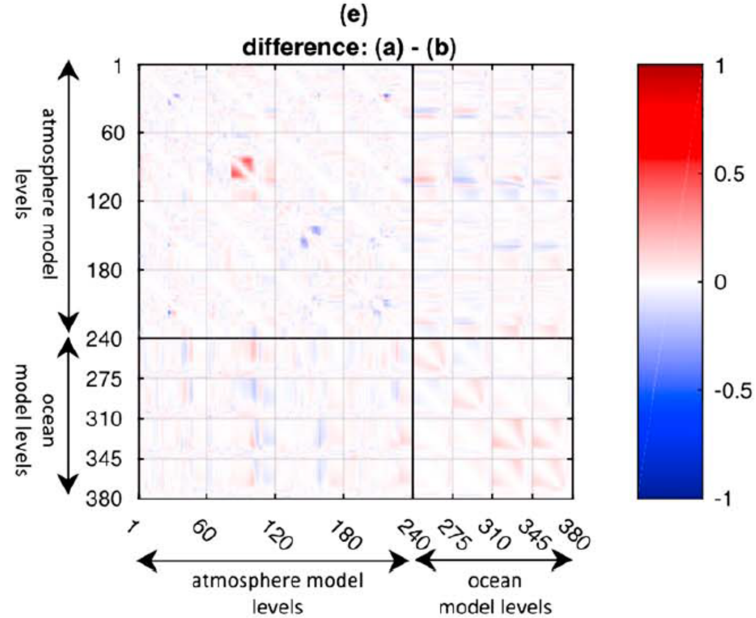
Localization – La=7500m, Lo=75m



Localized



Difference from original

Some sampling error removed, condition number $O(10^9)$.

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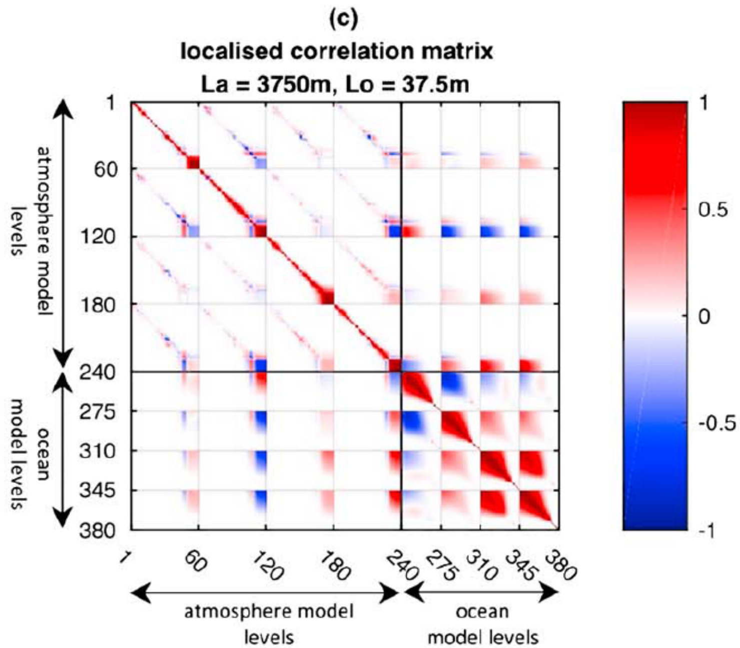
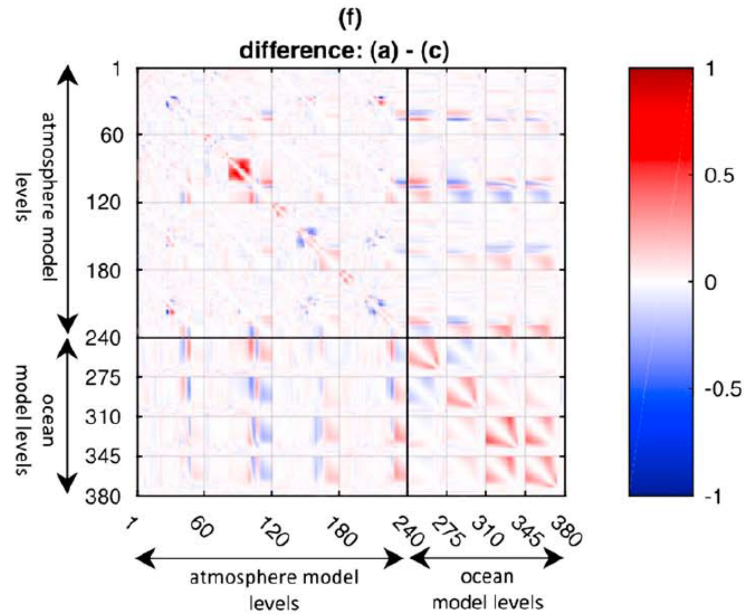
Localization – La=3750m, Lo=37.5m



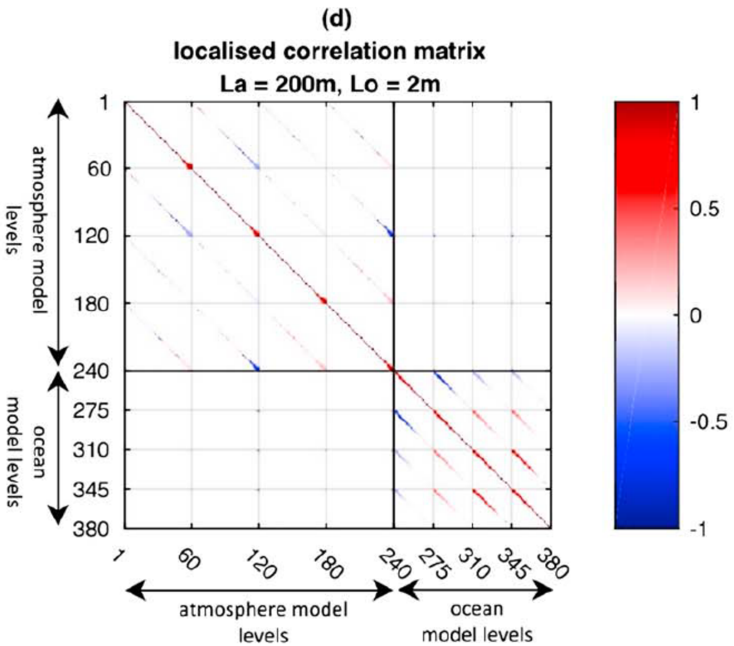
Localized



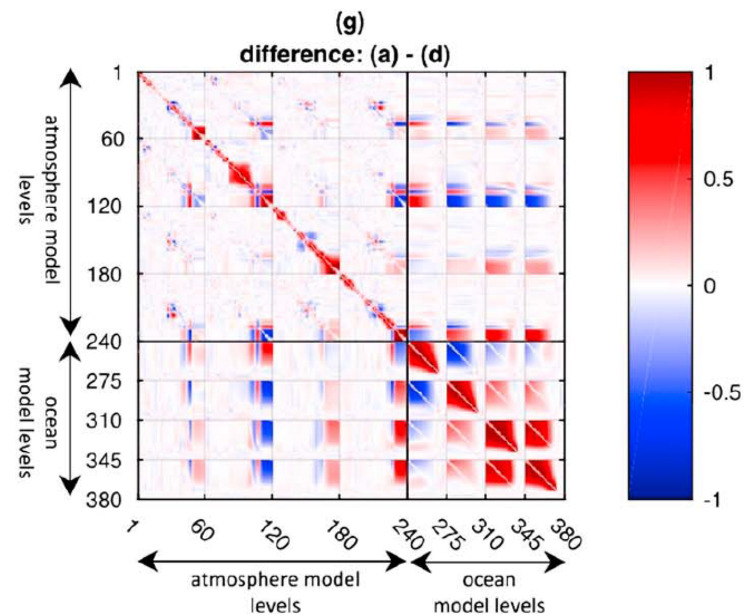
Difference from original

Halving length scales removed more sampling error removed, but strength of cross-correlations reduced. Condition number O(10^8).

Localization – La=200m, Lo=2m



Localized



Difference from original

To obtain condition number $O(10^4)$ we need to make lengthscales so short that correlations are destroyed.

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Reconditioning v localization

So far we have seen

- Reconditioning reduces condition number, but retains sampling noise.
- Localization reduces sampling noise, but matrix is still ill-conditioned.

➢ Can we obtain the best of both worlds by first reconditioning and then localizing?

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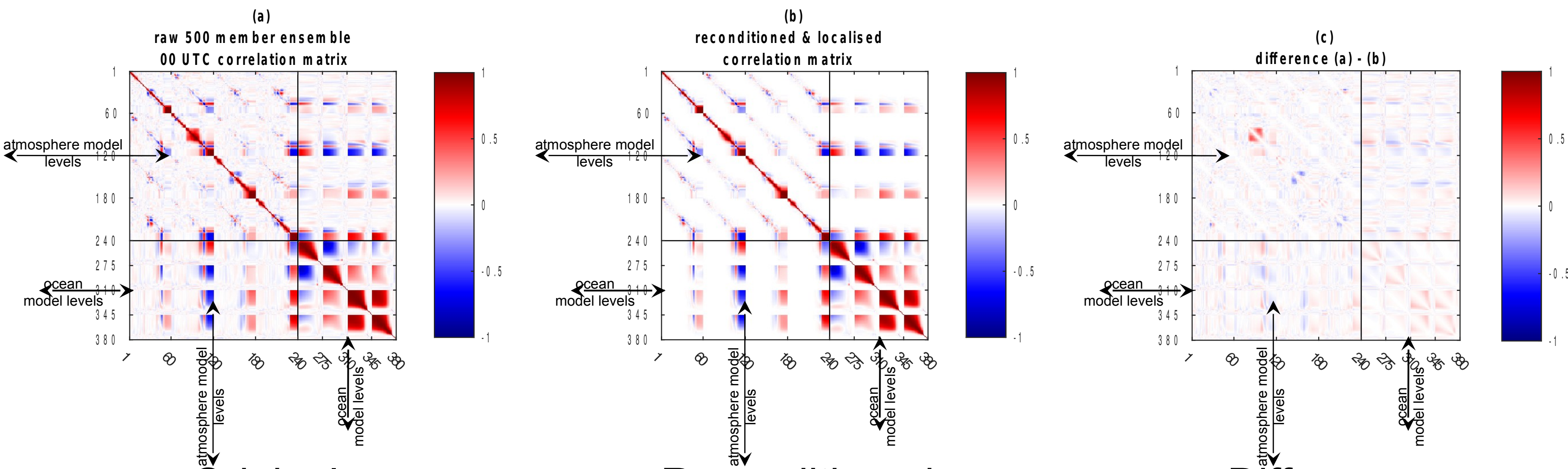
Reconditioning then localization



Original

Reconditioned then localized

Difference

- ✓ Sampling noise removed
- ✓ Cross-correlation signals retained
- ✓ Matrix is well conditioned – O(10^4)

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Summary

- Reconditioning of correlation matrix can reduce condition number, but sampling noise retained.
- Important to treat correlation matrix rather than covariance matrix.
- Localization can reduce sampling error, but matrix still ill-conditioned.
- Combination of both leads to a well-conditioned matrix with sampling error removed.

Smith, P.J., Lawless, A.S. and Nichols, N.K. (2018), Treating sample covariances for use in strongly coupled atmosphere-ocean data assimilation. *Geophysical Research Letters,* 45, doi: 10.1002/2017GL075534.

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