

# The accuracy of efficient particle filters

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# Efficient particle filters

1. Introduce localisation to reduce the number of observations, (not enough)
2. Approximations: Combine Particle Filters and Ensemble Kalman Filters or Gaussian Mixtures or second-order exact filters
3. Transportation
4. Use proposal-density freedom.

# 2-stage proposal

Introduce a 2-stage proposal:

1. For each  $i$  draw  $x_i^* \sim p(x^n | x_i^{n-1}, y^n)$

2. For each  $i$  draw  $\xi_i \sim N(0, P)$  with  $P^{-1} = Q^{-1} + H^T R^{-1} H$

3. For each  $i$  write  $x_i^n = x_i^* + \alpha_i P^{1/2} \xi_i$

4. Solve for  $\alpha_i$

$$w_i(\alpha_i) = \frac{p(y | x_i^{n-1}) p(x_i^n | x_i^{n-1}, y^n)}{q(x_i^n | x_i^* | x_{i;1:N}^{n-1}, y^n)} = w_{target}$$

# Limit for $N_x \rightarrow \infty$

In this limit the relation for  $\alpha_i$  reduces to

$$\alpha_i^2 = -\frac{\gamma_i}{N_x} W_{0,-1} \left[ -e^{c_i/N_x - 1} \right]$$

in which  $\gamma_i = \xi_i^T \xi_i \approx N_x \pm \sqrt{N_x}$  the size of random forcing,

and  $c_i \propto -\log [p(y^n | x_i^{n-1})] \propto \sqrt{N_y}$  optimal proposal weights,

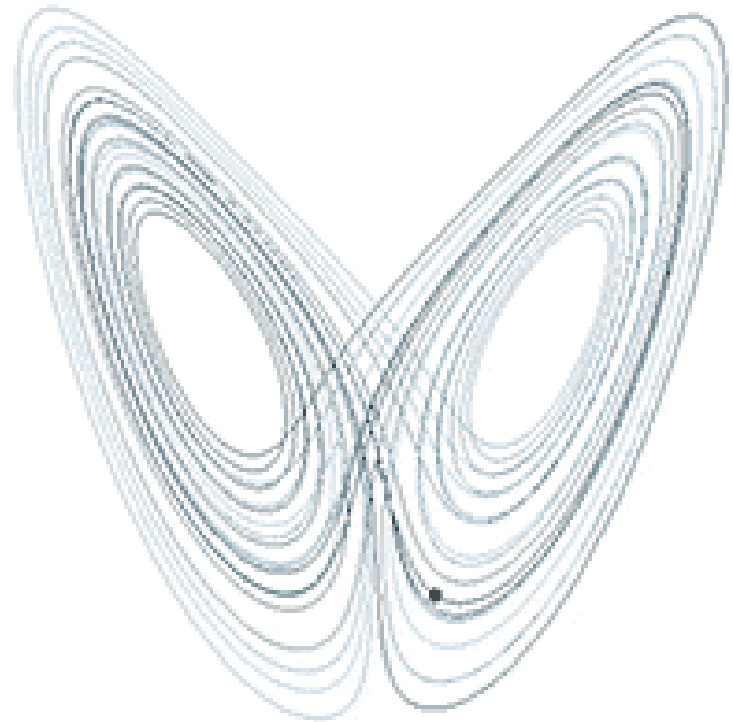
and  $W_{0,-1}$  the Lambert-W function with two branches '0' and '-1'.

Typically  $0.1 < \alpha < 3.0$

(Note, this is not the IEWPF)

# Experiments on Lorenz 1963 model

- 10,000 independent Lorenz 1963 models
- 30,000 variables, 10,000 parameters
- 10 particles
- Observations:
  - every 20 time steps,
  - first two variables
- Observation errors Gaussian
- SIR needs 500,000 particles for an effective ensemble size of about 300 on just one of the L63 models...



# Sequential parameter estimation

- SPDE 
$$x^n = f(x^{n-1}, \theta) + \beta^n$$

- Unknown parameter

$$x^n = f(x^{n-1}, \theta_0) + \frac{\partial f}{\partial \theta} (\theta - \theta_0) + \beta^n$$

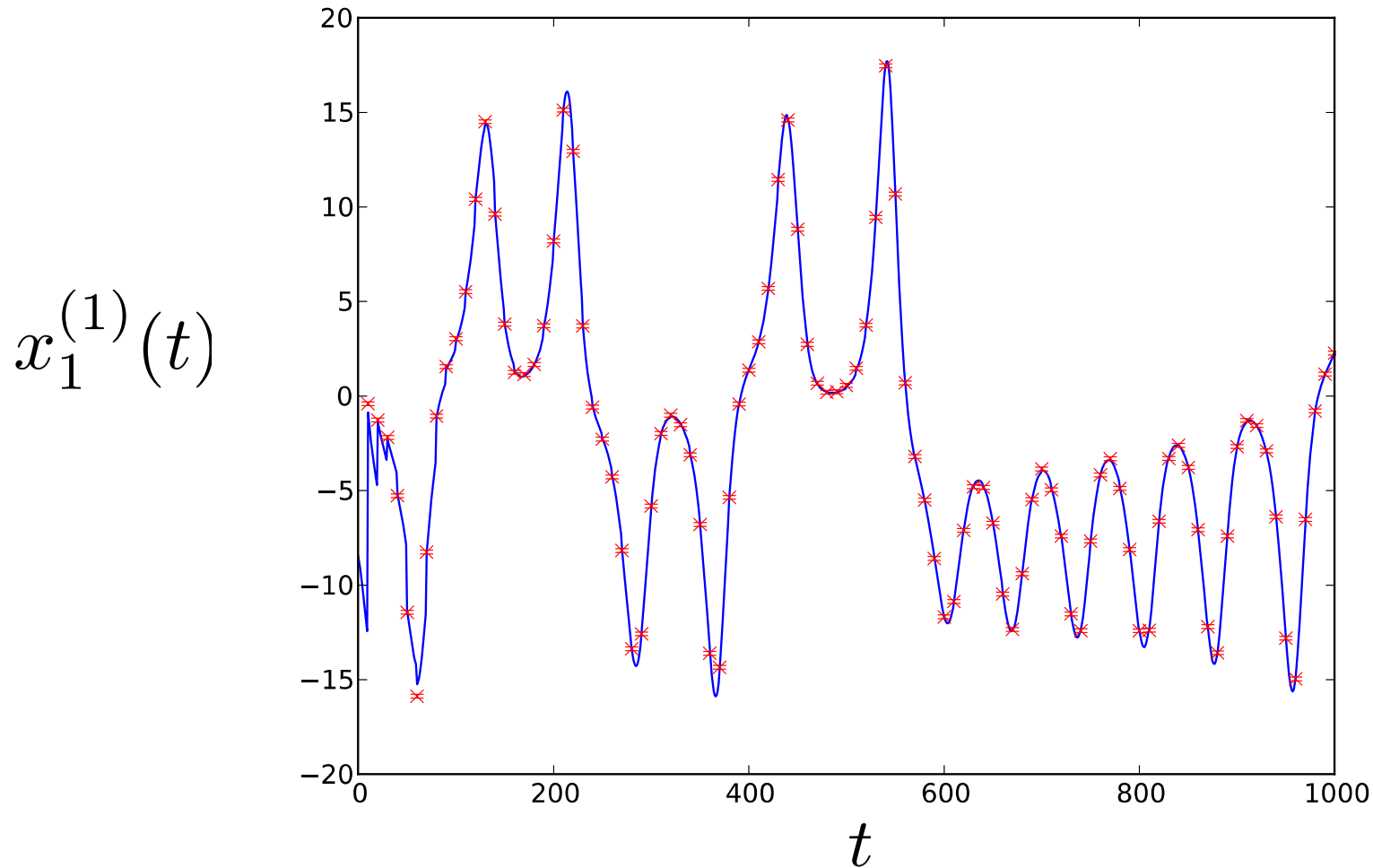
- Model as 
$$\theta^n = \theta^{n-1} + \eta^n$$

hence model error 
$$Q_{xx} = Q_\beta + \frac{\partial f}{\partial \theta} Q_\eta \frac{\partial f^T}{\partial \theta}$$

$$Q_{x\theta} = \frac{\partial f}{\partial \theta} Q_\eta$$

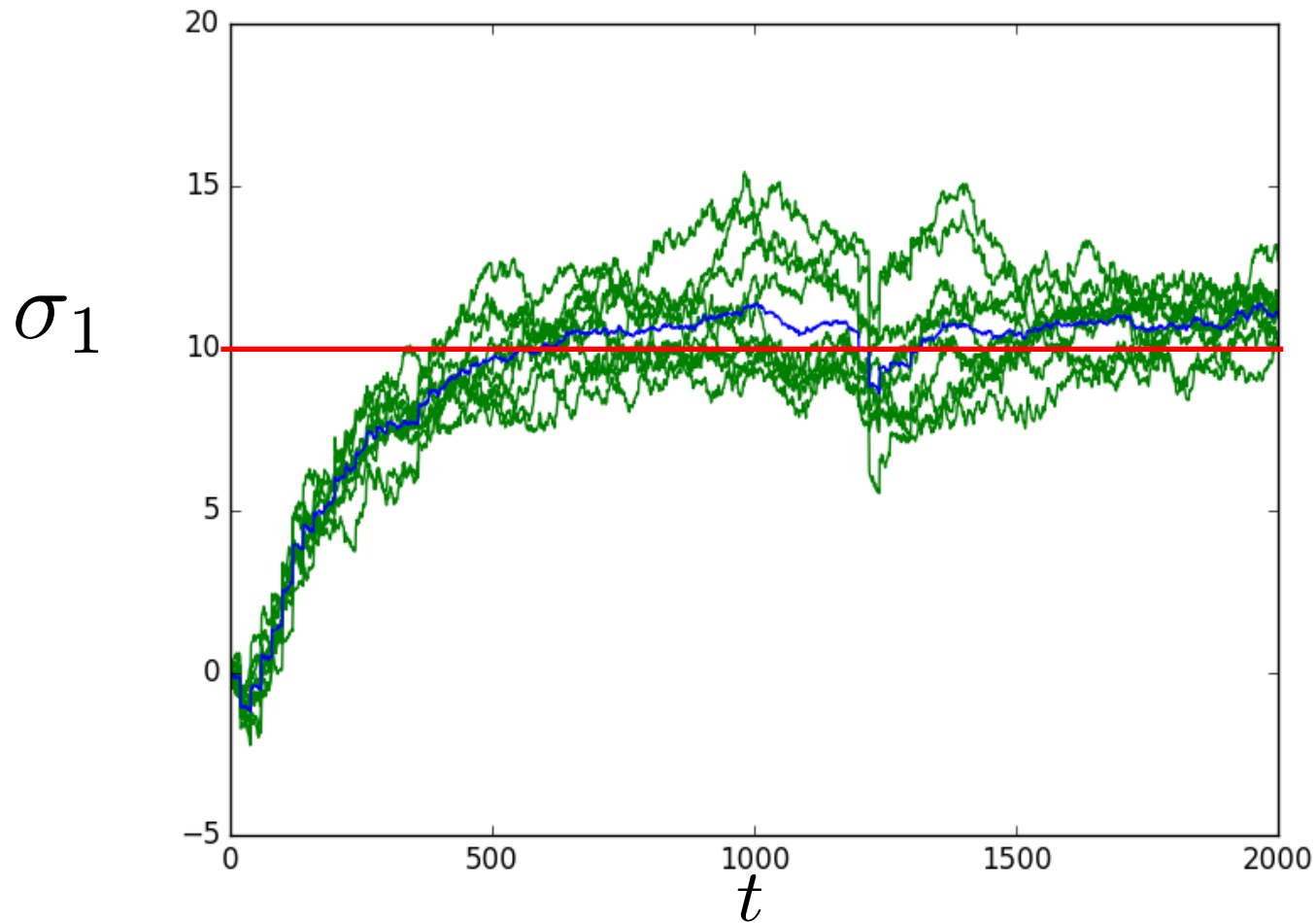
$$Q_{\theta\theta} = Q_\eta$$

# State evolution (one of 30,000)



Time evolution mean of first variable system 1, starting 10 lower than true value.

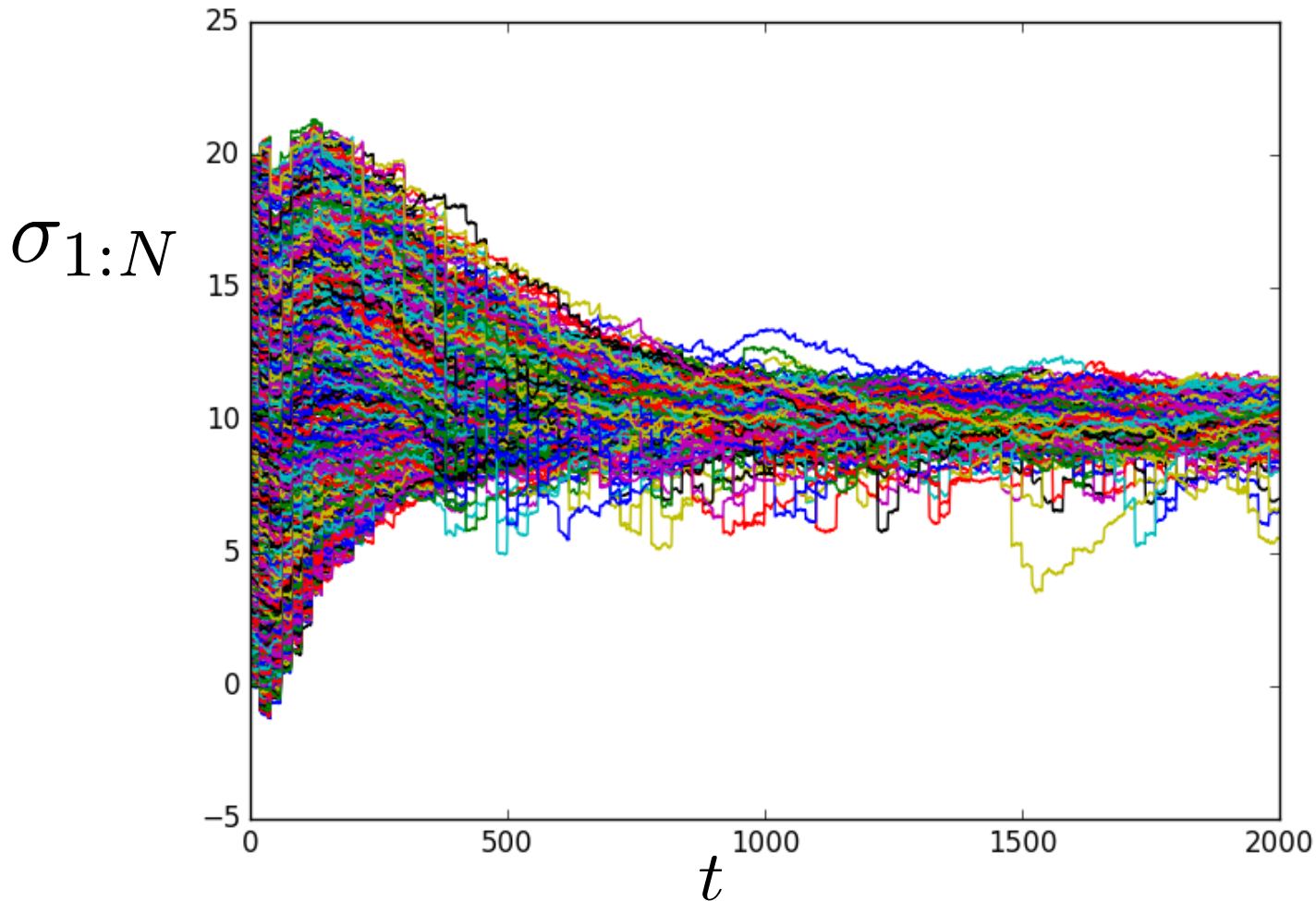
# Parameter evolution (one of 10,000)



Time evolution mean of parameter system 1, starting 10 lower than true value.

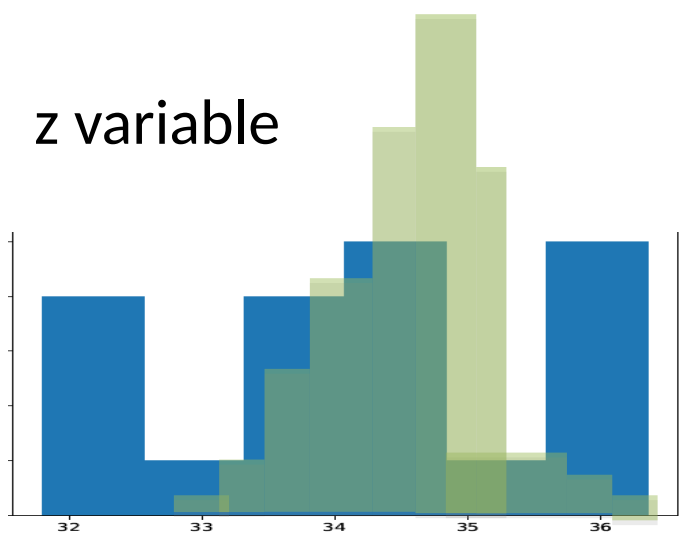
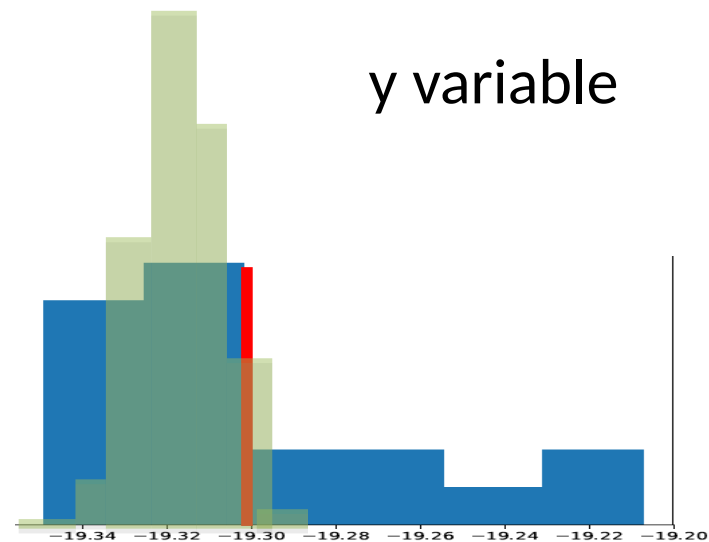
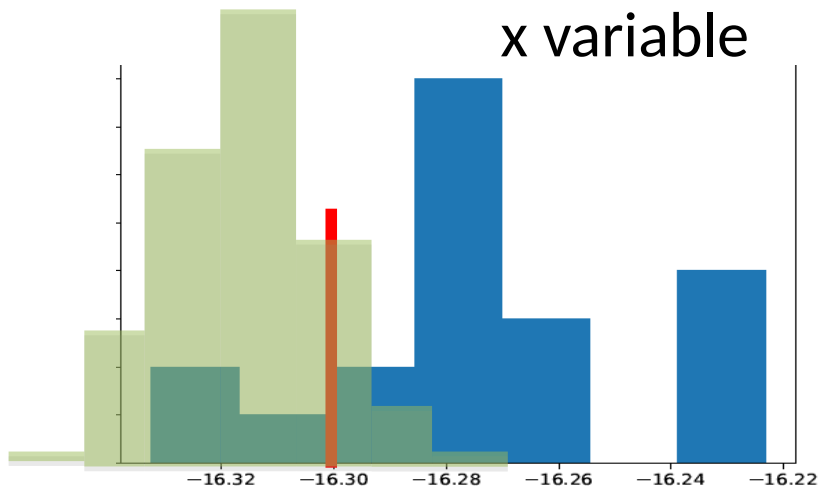


# Parameter mean values (dim=10,000)

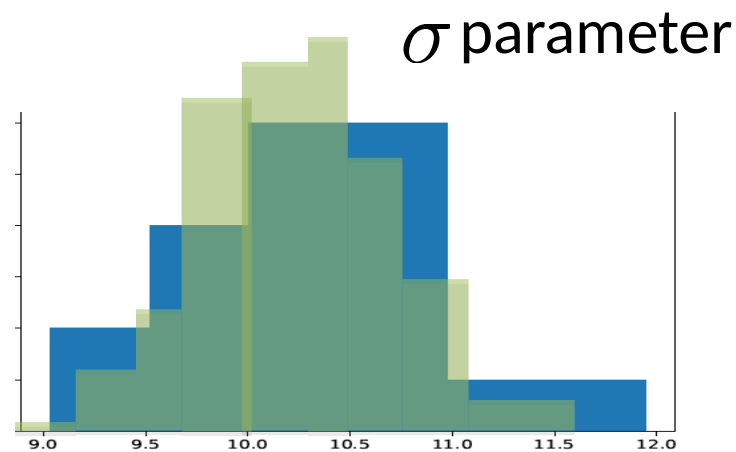


Time evolution mean values parameter all 10,000 systems, starting between 0 and 20.

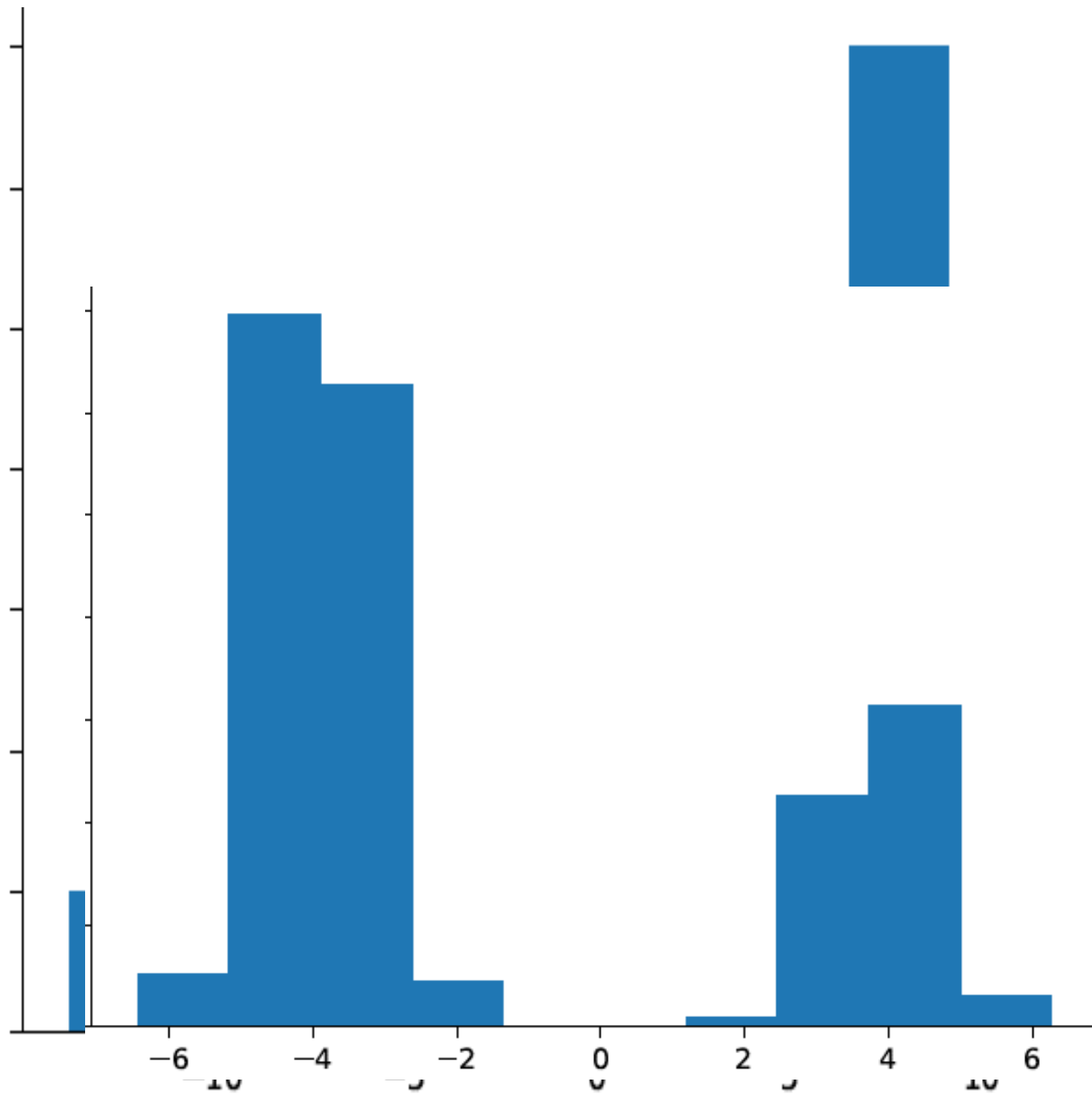
# Histogram system 1



$\theta_1$



Blue: Equal-weight PF 10 members, light green SIR 500,000 members



# Conclusions

- A fully **nonlinear non-degenerate** particle filter for systems with **high dimensions** has been derived.
- The filter can be viewed as an optimal proposal step to move particles followed by an equal-weight step.
- Taking the median as the target weight **might** mean the filter is unbiased/consistent.
- Pdfs with 10 members are not exact, but not nonsense
- **We need good estimate of Q...**

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Exascale Data Assimilation (50% U of Reading – 50% Met Office)

DARC/NCEO Data Assimilation (50% U of Reading – 50% NCEO)

Adverts out very soon, ask me for more details.

- Implicit Equal-weights Particle Filter Zhu, M, P.J. van Leeuwen, and J. Amezcu, Q J Royal Meteorol. Soc., doi: 10.1002/qj.2784, 2015
- [Particle filters for applications in the geosciences](#). Van Leeuwen, P.J., H. Kunsch, L. Nerger, R. Potthast, S. Reich, to be submitted to QJRMS.
- Nonlinear Data Assimilation. Van Leeuwen, P.J., Y. Cheng, and S. Reich., Springer, doi:10.1007/978-3-319-18347-3, 2015.

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