The accuracy of efficient particle filters

Peter Jan van Leeuwen

Javier Amezcua, Mengbin Zhu, Jacob Skauvold

Efficient particle filters

1. Introduce localisation to reduce the number of observations, (not enough)
2. Approximations: Combine Particle Filters and Ensemble Kalman Filters or Gaussian Mixtures or second-order exact filters
3. Transportation
4. Use proposal-density freedom.

2-stage proposal

Introduce a 2-stage proposal:

1. For each *i* draw $x_i^* \sim p(x^n | x_i^{n-1}, y^n)$

2. For each *i* draw $\xi_i \sim N(0, P)$ with $P^{-1} = Q^{-1} + H^T R^{-1} H$

3. For each *i* write $x_i^n = x_i^* + \alpha_i P^{1/2} \xi_i$

4. Solve for α_i in

$$w_i(\alpha_i) = \frac{p(y|x_i^{n-1}) p(x_i^n | x_i^{n-1}, y^n)}{q(x_i^n x_i^* | x_{i;1:N}^{n-1}, y^n)} = w_{target}$$

Limit for $N_x \to \infty$

In this limit the relation for α_i reduces to

$$\alpha_i^2 = -\frac{\gamma_i}{N_x} W_{0,-1}\left[-e^{c_i/N_x - 1}\right]$$

in which $\gamma_i = \xi_i^T \xi_i \approx N_x \pm \sqrt{N_x}$ the size of random forcing,

and $c_i \propto -log\left[p(y^n|x_i^{n-1})\right] \propto \sqrt{N_y}$ optimal proposal weights,

and $W_{0,-1}$ the Lambert-W function with two branches '0' and '-1'.

Typically $0.1 < \alpha < 3.0$

(Note, this is not the IEWPF)

Experiments on Lorenz 1963 model

- 10,000 independent Lorenz 1963 models
- 30,000 variables, 10,000 σ parameters
- 10 particles
- Observations:
  every 20 time steps,
  first two variables
- Observation errors Gaussian
- SIR needs 500,000 particles
  for an effective ensemble
  size of about 300 on just one
  of the L63 models…

Sequential parameter estimation

- SPDE $$x^n = f(x^{n-1}, \theta) + \beta^n$$

- Unknown parameter
$$x^n = f(x^{n-1}, \theta_0) + \frac{\partial f}{\partial \theta}(\theta - \theta_0) + \beta^n$$

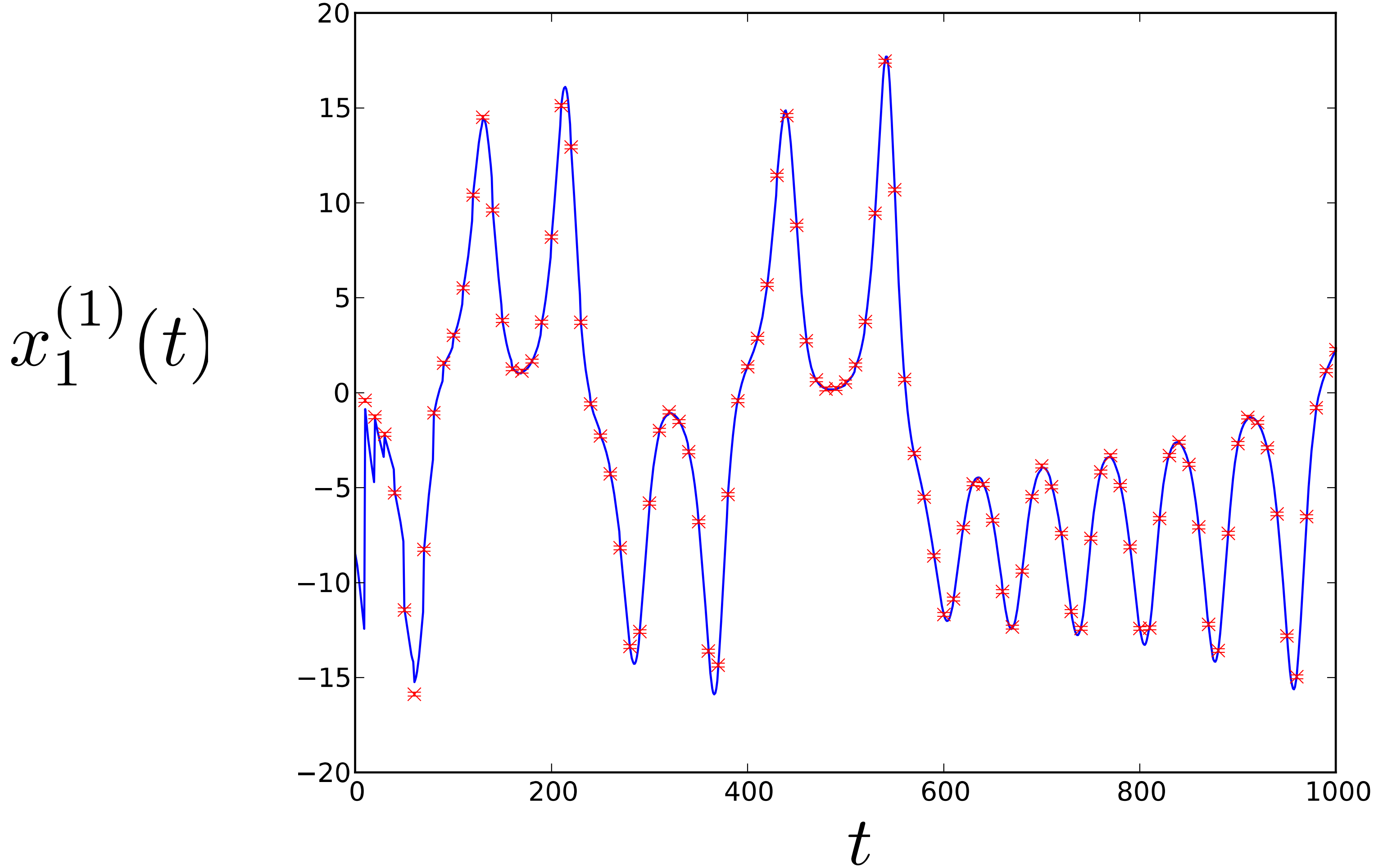
- Model as $$\theta^n = \theta^{n-1} + \eta^n$$

hence model error $$Q_{xx} = Q_\beta + \frac{\partial f}{\partial \theta} Q_\eta \frac{\partial f}{\partial \theta}^T$$

$$Q_{x\theta} = \frac{\partial f}{\partial \theta} Q_\eta$$

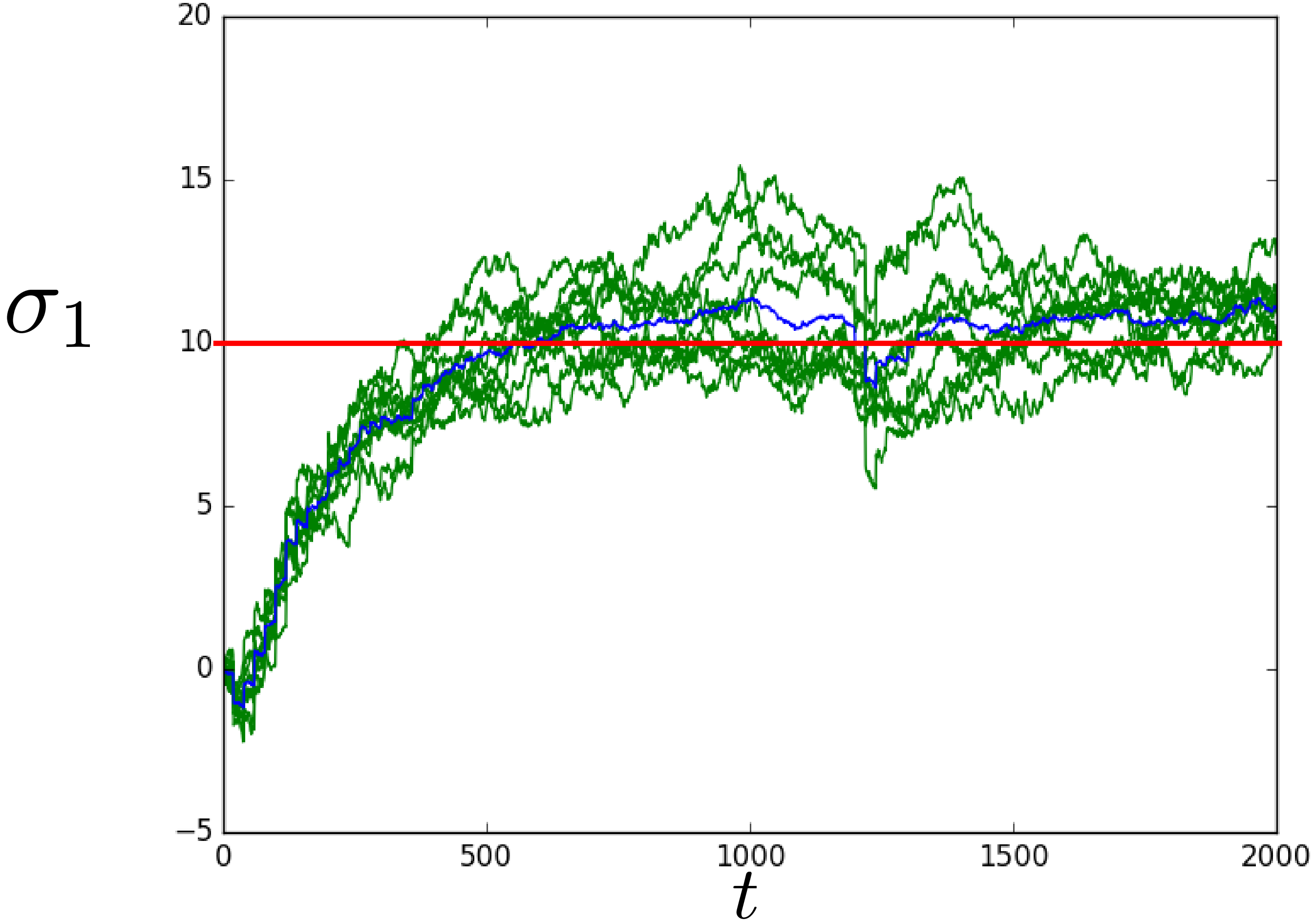
$$Q_{\theta\theta} = Q_\eta$$

State evolution (one of 30,000)



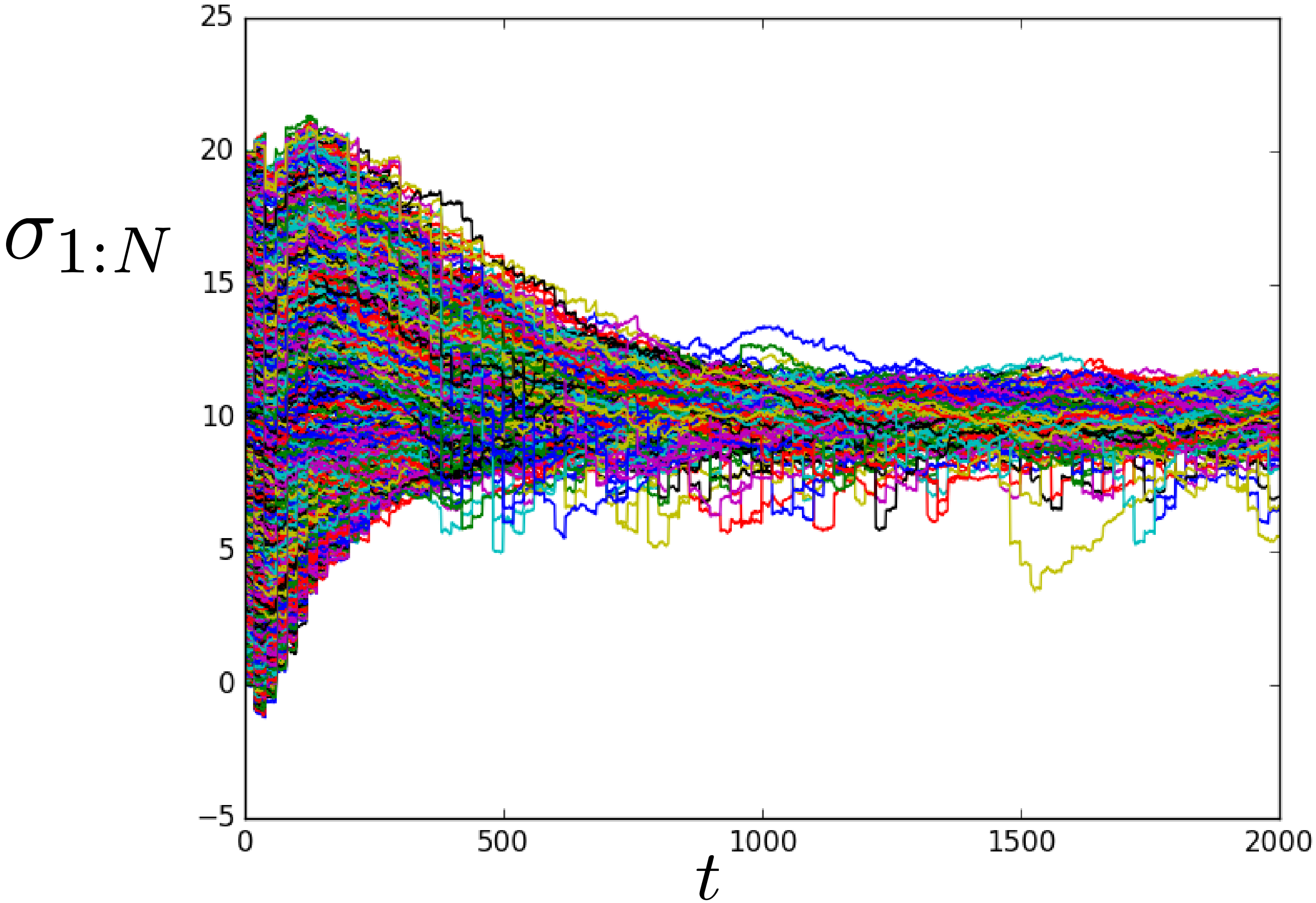
Time evolution mean of first variable system 1, starting 10 lower than true value.

Parameter evolution (one of 10,000)



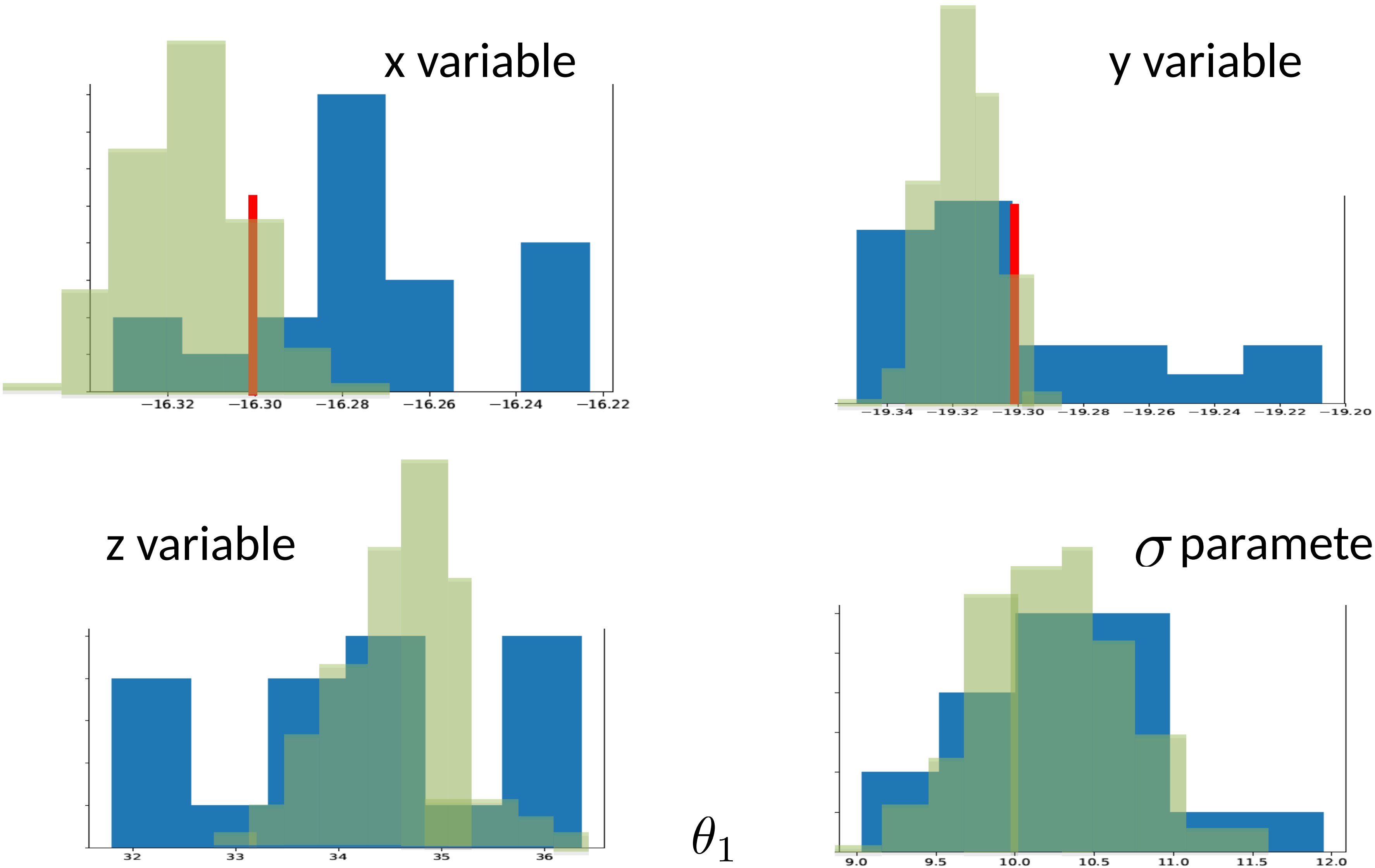
Time evolution mean of parameter system 1, starting 10 lower than true value.

Parameter mean values (dim=10,000)

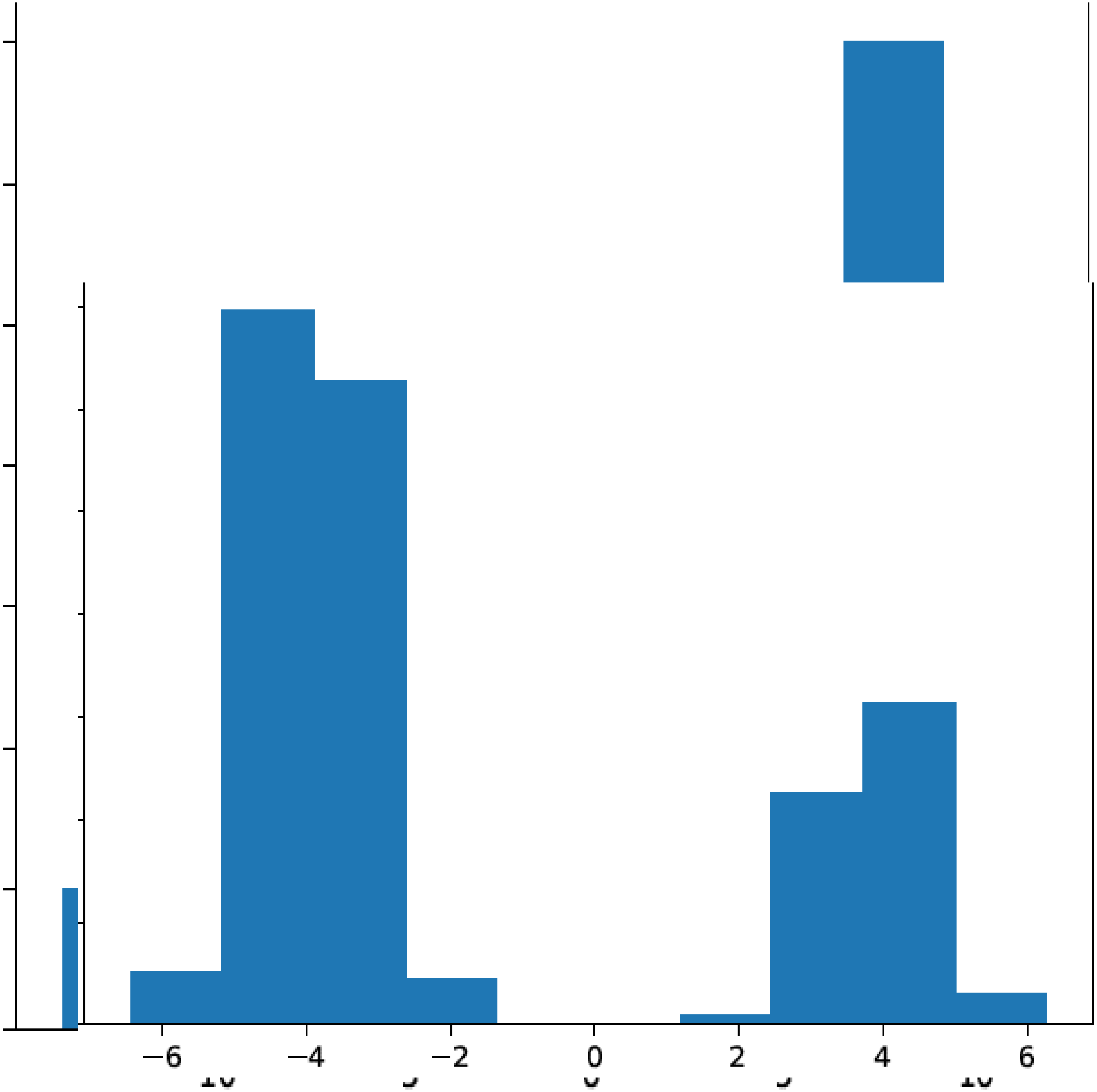


Time evolution mean values parameter all 10,000 systems, starting between 0 and 20.

Histogram system 1



Blue: Equal-weight PF 10 members, light green SIR 500,000 members



−6
−4
−2
0
2
4
6

Conclusions

- A fully nonlinear non-degenerate particle filter for systems with high dimensions has been derived.
- The filter can be viewed as an optimal proposal step to move particles followed by an equal-weight step.
- Taking the median as the target weight might mean the filter is unbiased/consistent.
- Pdfs with 10 members are not exact, but not nonsense
- We need good estimate of Q…

Two new full professorship positions at the University of Reading:

Exascale Data Assimilation (50% U of Reading – 50% Met Office)

DARC/NCEO Data Assimilation (50% U of Reading – 50% NCEO)

Adverts out very soon, ask me for more details.

- Implicit Equal-weights Particle Filter Zhu, M, P.J. van Leeuwen, and J. Amezcua, Q J Royal Meteorol. Soc., doi: 10.1002/qj.2784, 2015
- Particle filters for applications in the geosciences. Van Leeuwen, P.J.,H. Kunsch, L. Nerger, R. Potthast, S. Reich, to be submitted to QJRMS.
- Nonlinear Data Assimilation. Van Leeuwen, P.J., Y. Cheng, and S. Reich., Springer, doi:10,1007/978-3-319-18347-3, 2015.

This project has received funding from the European Research Council (ERC) under the European Union’s Horizon 2020 research and innovation programme.