Nonlinear effects in the ECMWF 4D-Var

Massimo Bonavita, Peter Lean and Elias Holm

ECMWF, Shinfield Park, RG2 9AX, Reading, UK

Outline

- Nonlinear effects: how important are they?
- Dealing with nonlinearity
- Implications for DA strategy

Nonlinear effects

- Non-linear and non-Gaussian effects are linked topics: model and observation operator nonlinearities inevitably produce non-Gaussian priors etc.
- We focus here mainly on nonlinear effects, but important to keep connection in mind
- Subject of many studies in the past (Andersson et al., 2005; Trémolet, 2005; Radnóti et al., 2005)
- Worth revisiting in light of much higher resolution of model and analysis and vastly increased use of non-linear observations (e.g., all-sky radiances)

Nonlinear effects

- Nonlinear strong constraint 4D-Var:

$$J(\mathbf{x}_0) = \frac{1}{2}(\mathbf{x}_0 - \mathbf{x}_b)^{\mathrm{T}}\mathbf{P}_b^{-1}(\mathbf{x}_0 - \mathbf{x}_b) + \frac{1}{2}\sum_{k=0}^{K}\left(\mathbf{y}_k - G_k(\mathbf{x}_0)\right)^{\mathrm{T}}\mathbf{R}_k^{-1}\left(\mathbf{y}_k - G_k(\mathbf{x}_0)\right)$$

where $G_k = H_k \circ M_{t_0 \to t_k}$ is a generalised observation operator that includes model propagation

- Incremental strong constraint 4D-Var approximates nonlinear cost function as a sequence of minimizations of linear, quadratic cost functions defined in terms of perturbations $\delta\mathbf{x}_0$ around a sequence of progressively more accurate trajectories $\mathbf{x}^t$:

$$J(\delta\mathbf{x}_0) = \frac{1}{2}(\delta\mathbf{x}_0 + \mathbf{x}_0^t - \mathbf{x}_b)^{\mathrm{T}}\mathbf{P}_b^{-1}(\delta\mathbf{x}_0 + \mathbf{x}_0^t - \mathbf{x}_b) + \frac{1}{2}\sum_{k=0}^{K}\left(\mathbf{d}_k - \mathbf{G}_k(\delta\mathbf{x}_0)\right)^{\mathrm{T}}\mathbf{R}_k^{-1}\left(\mathbf{d}_k - \mathbf{G}_k(\delta\mathbf{x}_0)\right)$$

where $\mathbf{G}_k = \mathbf{H}_k\mathbf{M}_{t_0\to t_k} = \mathbf{H}_k\mathbf{M}_{t_{k\_1}\to t_k}\mathbf{M}_{t_{k\_2}\to t_{k\_1}} \ldots \mathbf{M}_{t_0\to t_1}$ is the linearised version of G_k and $\mathbf{d}_k = \mathbf{y}_k - G_k(\mathbf{x}_0^t)$ are the observation departures around the latest model trajectory

Nonlinear effects

- The incremental formulation is advantageous for many reasons:
  1. Computational cost through the use of lower resolution linearised models
  2. Quadratic cost function guarantees convergence and uniqueness of the minimisation
  3. Quadratic cost function allows use of efficient gradient-based minimisers
- Going from nonlinear to incremental formulation requires the tangent linear approx.:

$$\mathbf{y}_k - G_k(\mathbf{x}_0) = \mathbf{y}_k - G_k(\mathbf{x}_0^t + \delta\mathbf{x}_0) = \mathbf{y}_k - G_k(\mathbf{x}_0^t) - \mathbf{G}_k(\delta\mathbf{x}_0) - \frac{1}{2}(\delta\mathbf{x}_0)^{\mathrm{T}}\left(\frac{\partial \mathbf{G}_k}{\partial \mathbf{x}}\right)_{\mathbf{x}^t}(\delta\mathbf{x}_0) - O(\|\delta\mathbf{x}_0\|^3) \approx$$

$$\mathbf{y}_k - G_k(\mathbf{x}_0^t) - \mathbf{G}_k(\delta\mathbf{x}_0)$$

- The TL approximation implies either or both:
  1. Small increments (when scaled w.r.to observation errors);
  2. Small sensitivity of $\mathbf{G}_k$=$\mathbf{H}_k\mathbf{M}_{t_0\to t_k}$ to linearization trajectory => approx. linear H and M

Model nonlinearities

- Model non-linearity: how much initial increments evolved by the linearised model and the nonlinear model differ?

$$M(\mathbf{x}^t + \delta\mathbf{x}_0) \approx M(\mathbf{x}^t) + \mathbf{M}(\delta\mathbf{x}_0)$$

Model nonlinearities

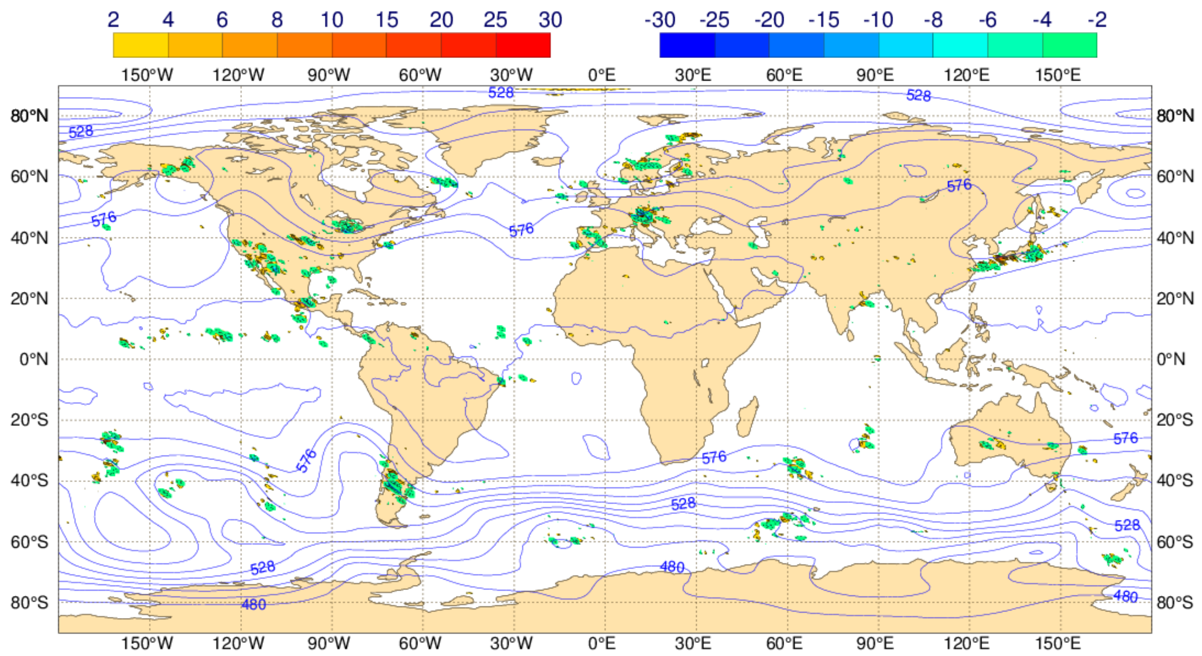
- Model non-linearity: how much initial increments evolved by the linearised model and the nonlinear model differ?

$$stdev\left(M(\mathbf{x}^t + \delta\mathbf{x}_0) - \left(M(\mathbf{x}^t) + \mathbf{M}(\delta\mathbf{x}_0)\right)\right)$$

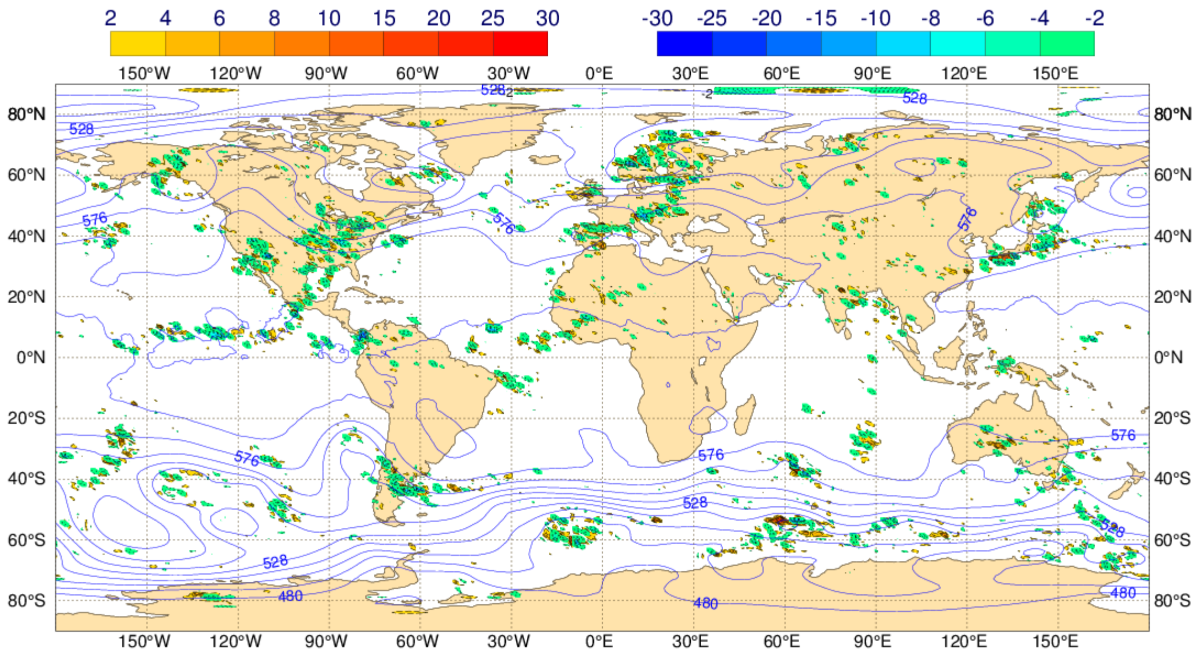
Vorticity 500 hPa (units: $10^{-5}s^{-1}$)

4D-Var, TCo639 outer loops, TL191/191 inner loops

T+3h



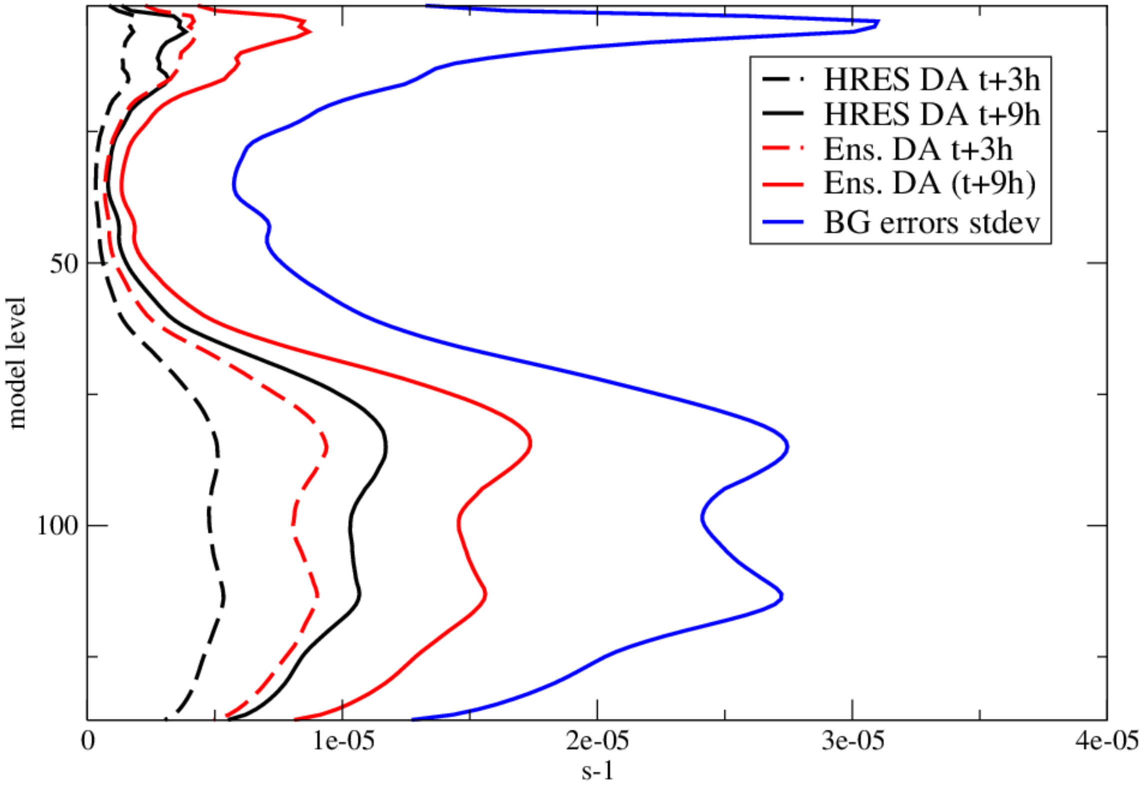
T+9h



Model nonlinearities

$$StDev\left(M(\mathbf{x}^t + \delta\mathbf{x}_0) - \left(M(\mathbf{x}^t) + \mathbf{M}(\delta\mathbf{x}_0)\right)\right)$$

Vorticity (units: $10^{-5}s^{-1}$)

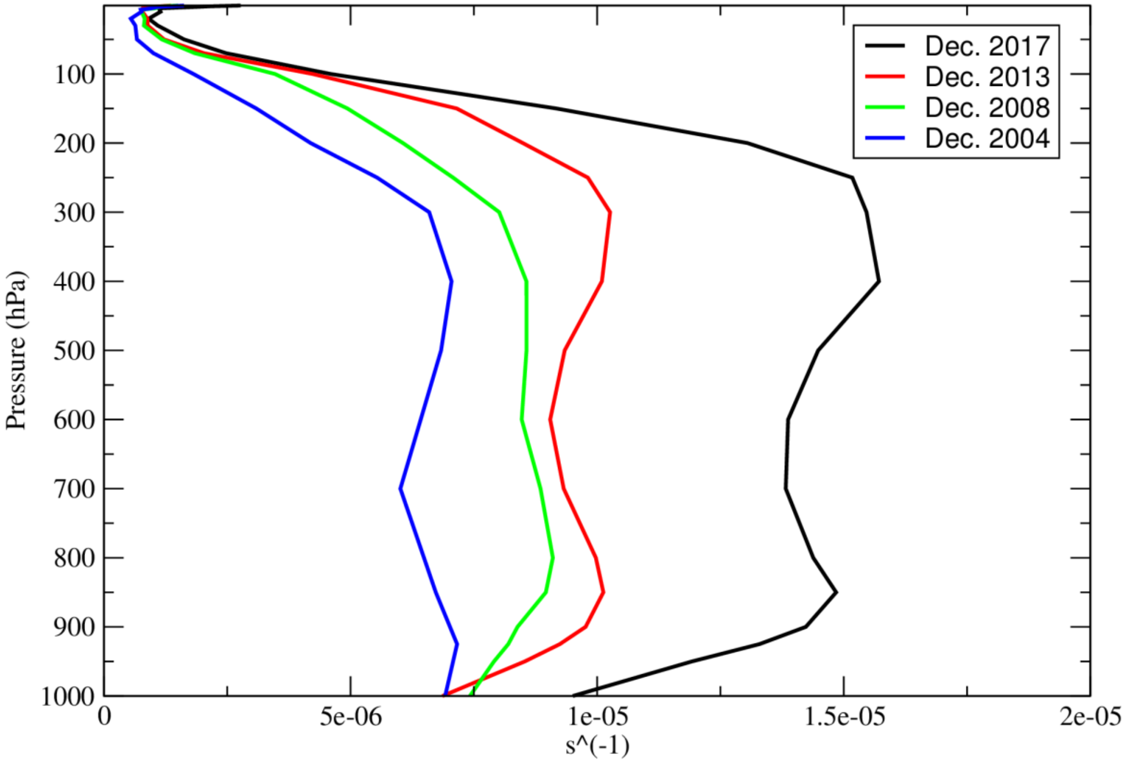


- Predominant in the Troposphere, negligible in Stratosphere
- 10 to 50% of size of background errors
- Rapid increase of nonlinearity with length of assimilation window

Model nonlinearities

$$StDev\left(M(\mathbf{x}^t + \delta\mathbf{x}_0)\ - \left(M(\mathbf{x}^t) + \mathbf{M}(\delta\mathbf{x}_0)\right)\right)$$

Vorticity (units: $10^{-5}s^{-1}$)



- Larger nonlinearities than in the past, due to:

1. Increased resolution (40 km -> 9 km)
2. Increase ratio of outer-inner loop resol. (3->5)
3. Less diffusive model

Observation nonlinearities

- Non-linearity from observation operators

- For linear observation operators in an ensemble DA:

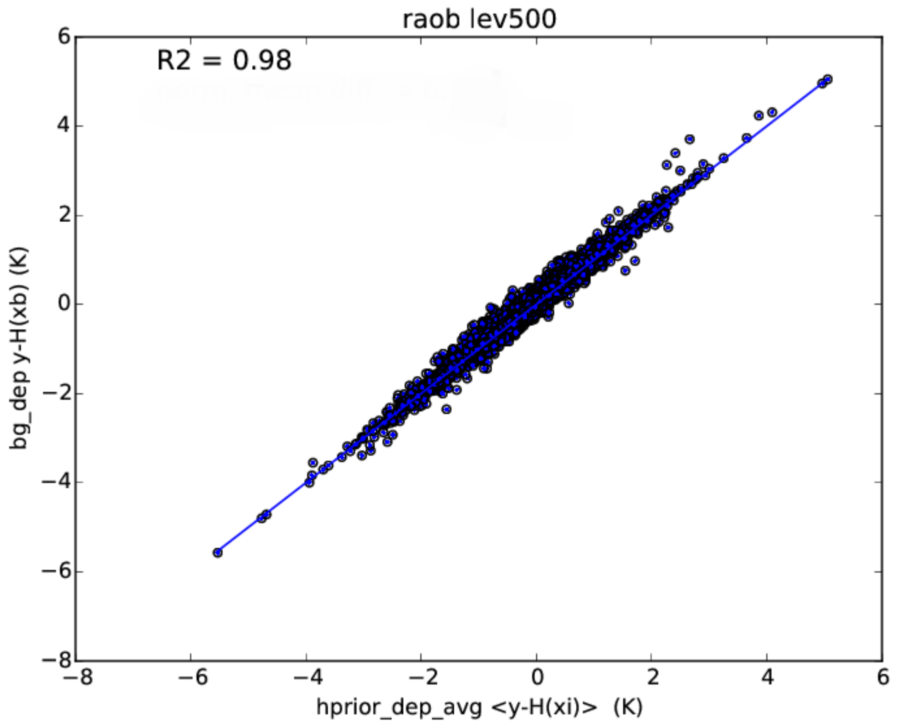
$$H\left(\mathbf{x}_b^{ctrl}\right) = H\left(M\left(\langle\mathbf{x}_a^i\rangle\right)\right) \approx \langle H\left(\mathbf{x}_b^i\right)\rangle \quad i = 1, \dots, N_{ens}$$

Observation nonlinearities

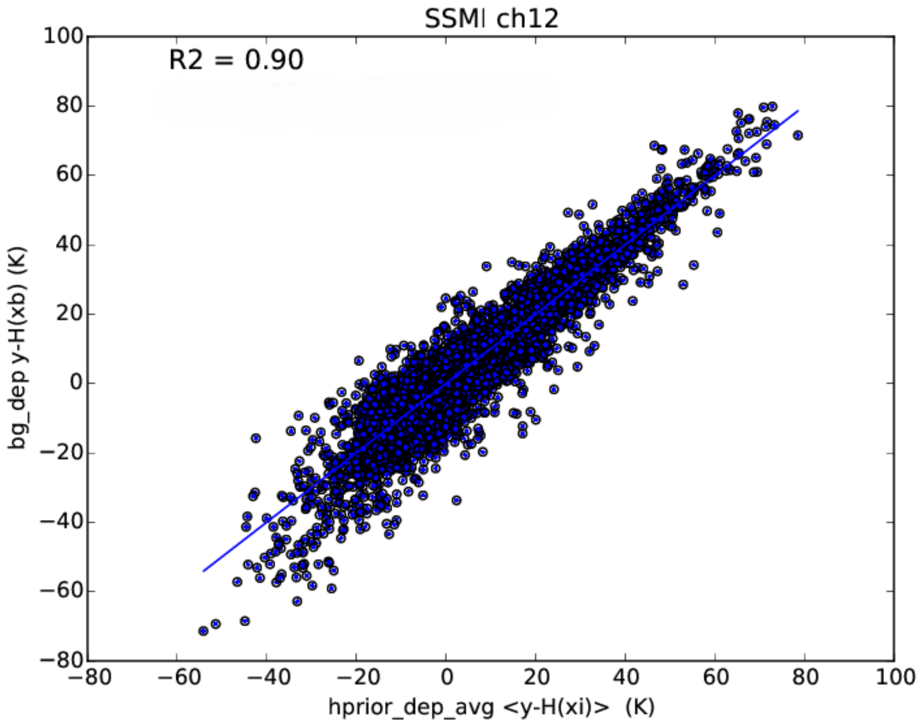
- For linear observation operators in an ensemble DA:

$$H\left(\mathbf{x}_b^{ctrl}\right) \approx \left\langle H\left(\mathbf{x}_b^i\right)\right\rangle \quad i = 1, \dots, N_{ens}$$

Radiosonde Temp. 500 hPa



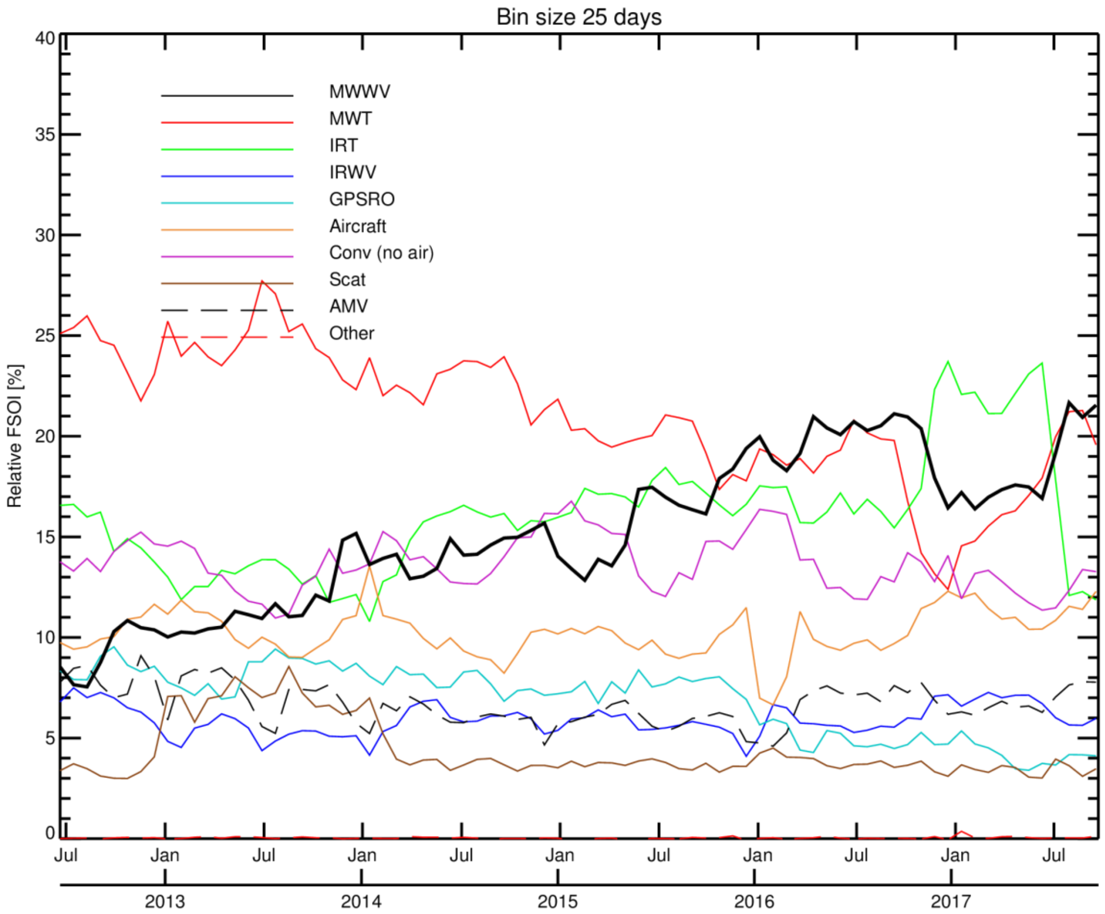
Mwave imager, rain and cloud



Observation nonlinearities

Nonlinear effects in 4D-Var become increasingly important with **ever increasing influence of nonlinear observations**

Forecast sensitivity (FSO) of major observing systems in ECMWF DA



from Alan Geer

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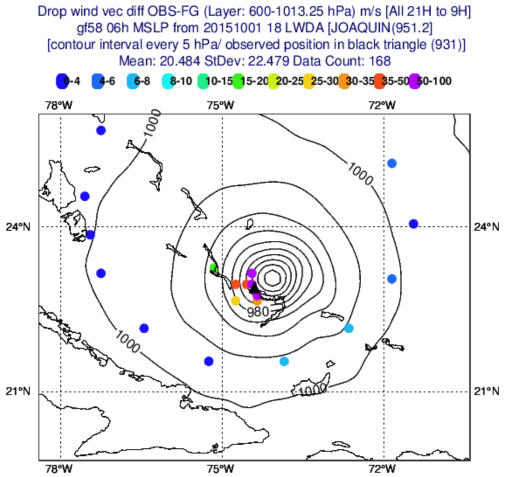
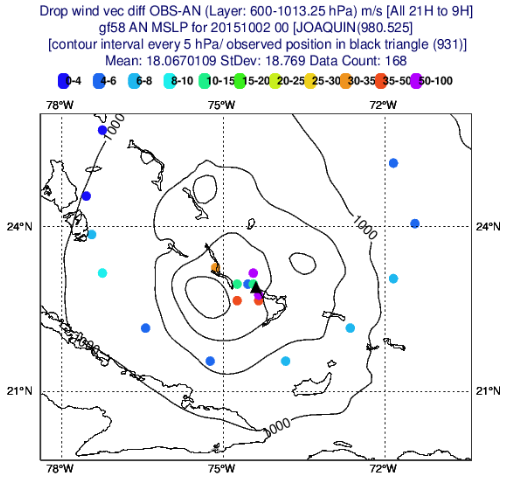
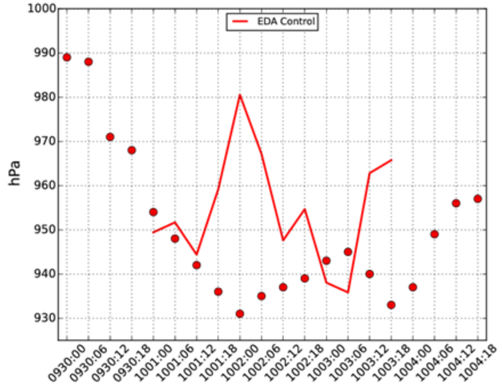
Nonlinear effects: large increments

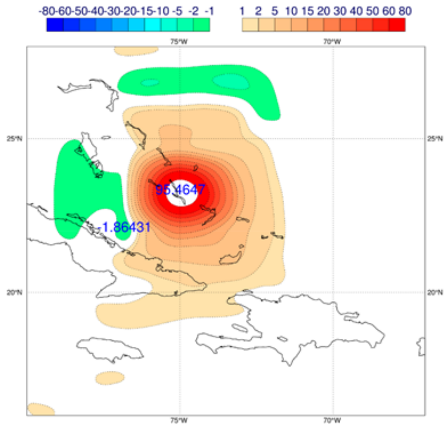
- When does the incremental approach break down?





Tropical Cyclone Joaquin, 2015-10-02 00UTC
Near TC core O-B wind departures 30-80 m/s!

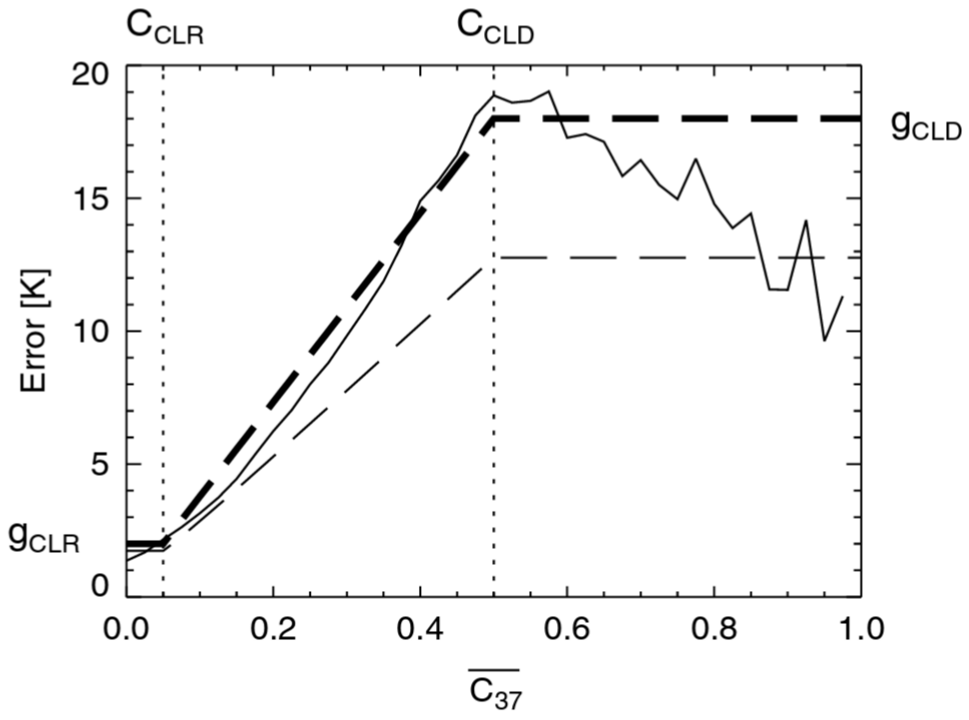




Bonavita et al., 2017: On the initialization of Tropical Cyclones. ECMWF Tech. Memo. n. 810.

Nonlinear effects: large increments

- Remedy: reduce increments by increase of prescribed Observation Errors (taking representativeness error into account)

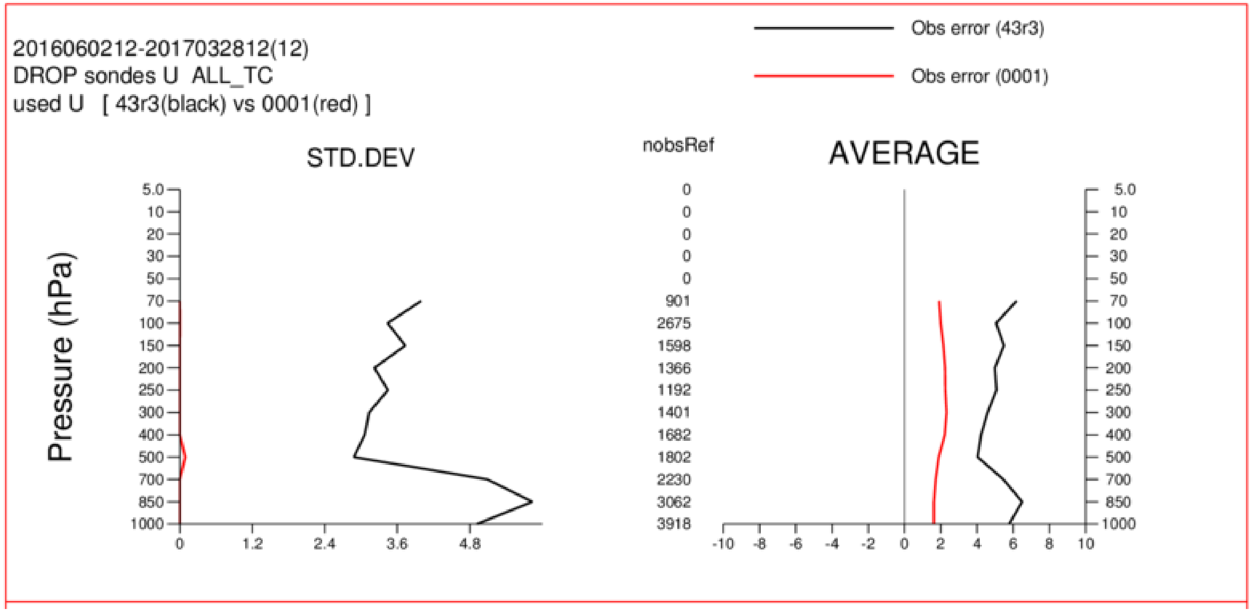


Observation error model for AMSRE ch.19v all-sky radiances as a function of “symmetric” (forecast and observed) cloud amount (Geer and Bauer, 2011)

Nonlinear effects: large increments

- Remedy: reduce increments by increase of prescribed Observation Errors (taking representativeness error into account)

$$\left\langle \left(y - G\left(\mathbf{x}_0^b\right)\right)^2 \right\rangle = \sigma_b^2 + \sigma_o^2 = \sigma_b^2 + \sigma_{o,I}^2 + \sigma_{o,R}^2 + \sigma_{o,H}^2$$

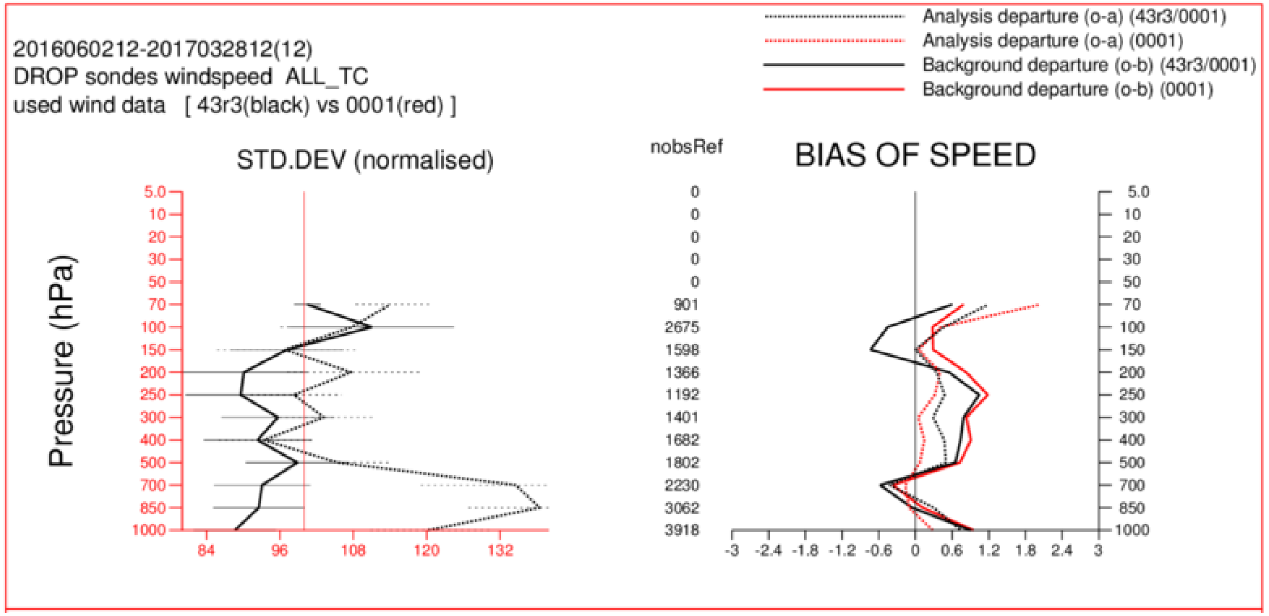


Observation error model for dropsondes winds (Bonavita et al., 2017)

Nonlinear effects: large increments

- Remedy: reduce increments by increase of prescribed Observation Errors (taking representativeness error into account)

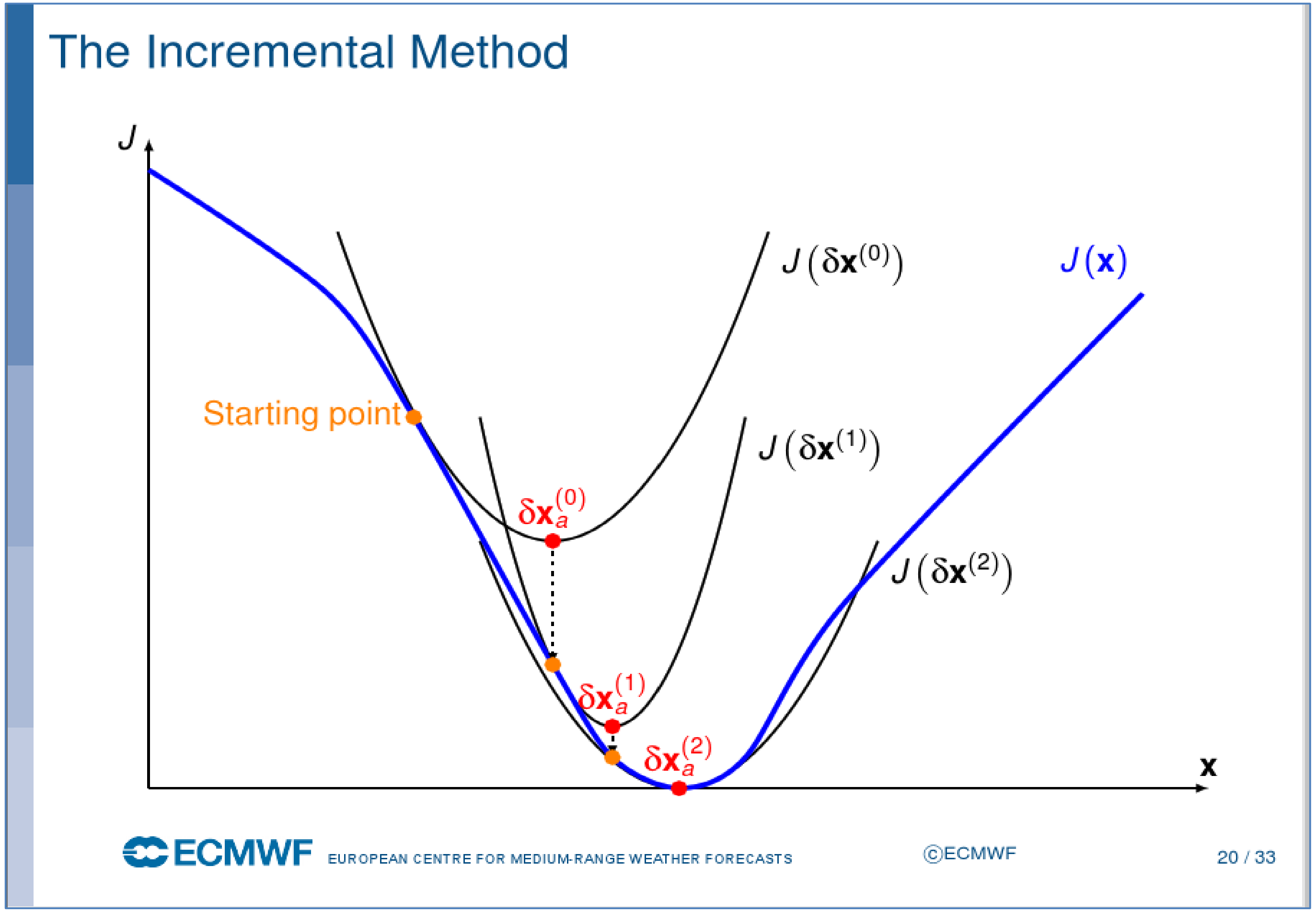
$$\left\langle \left(y - G\left(\mathbf{x}_0^b\right)\right)^2 \right\rangle = \sigma_b^2 + \sigma_o^2 = \sigma_b^2 + \sigma_{o,I}^2 + \sigma_{o,R}^2 + \sigma_{o,H}^2$$



Observation departures statistics for dropsondes wind speed (Bonavita et al., 2017)

Nonlinear effects: the incremental approach

- Incremental 4D-Var deals with nonlinearities by a succession of linear optimizations around progressively more accurate first guess trajectories:



S. Massart

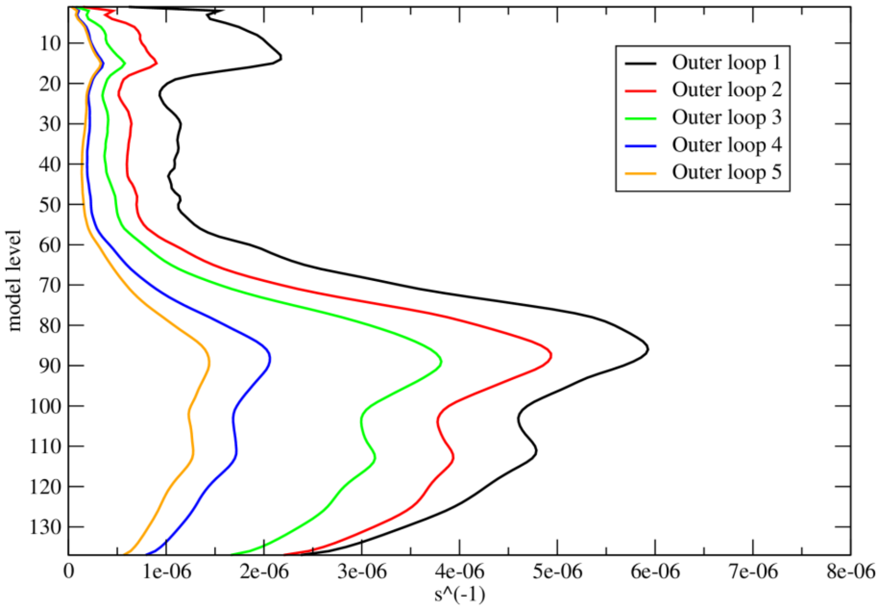
ECMWF

Nonlinear effects: the incremental approach

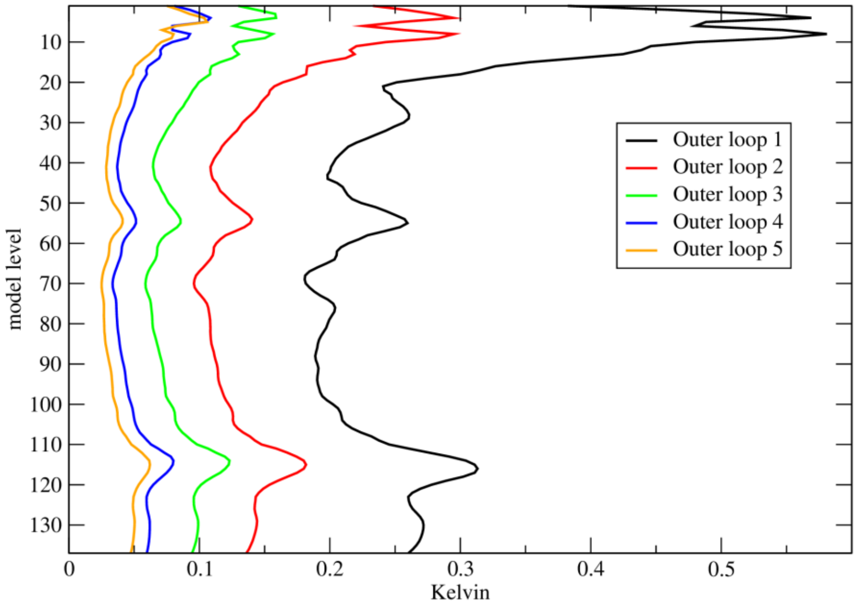
- Incremental 4D-Var deals with nonlinearities by a succession of linear optimizations around progressively more accurate first guess trajectories

- Successive linearization trajectories should get progressively closer to the solution of the nonlinear cost function, increments should get progressively smaller and the tangent liner hypothesis becomes gradually more valid

Vorticity analysis incr.



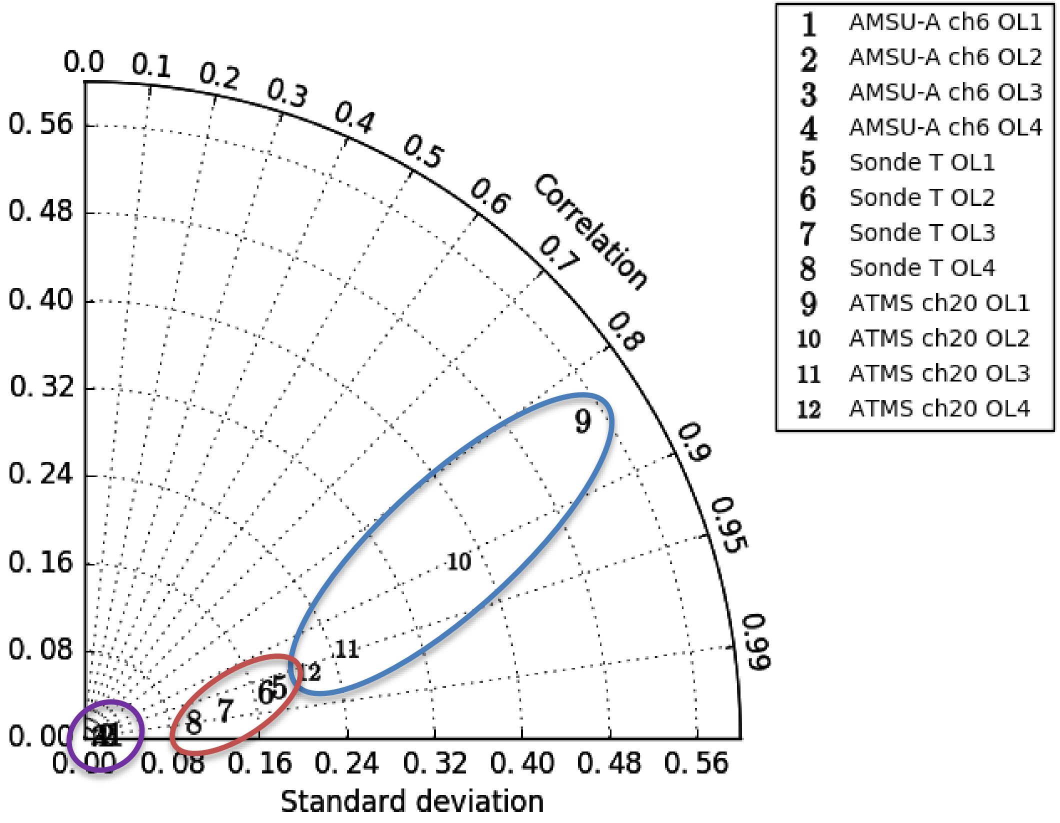
Temperature analysis incr.



ORECAST

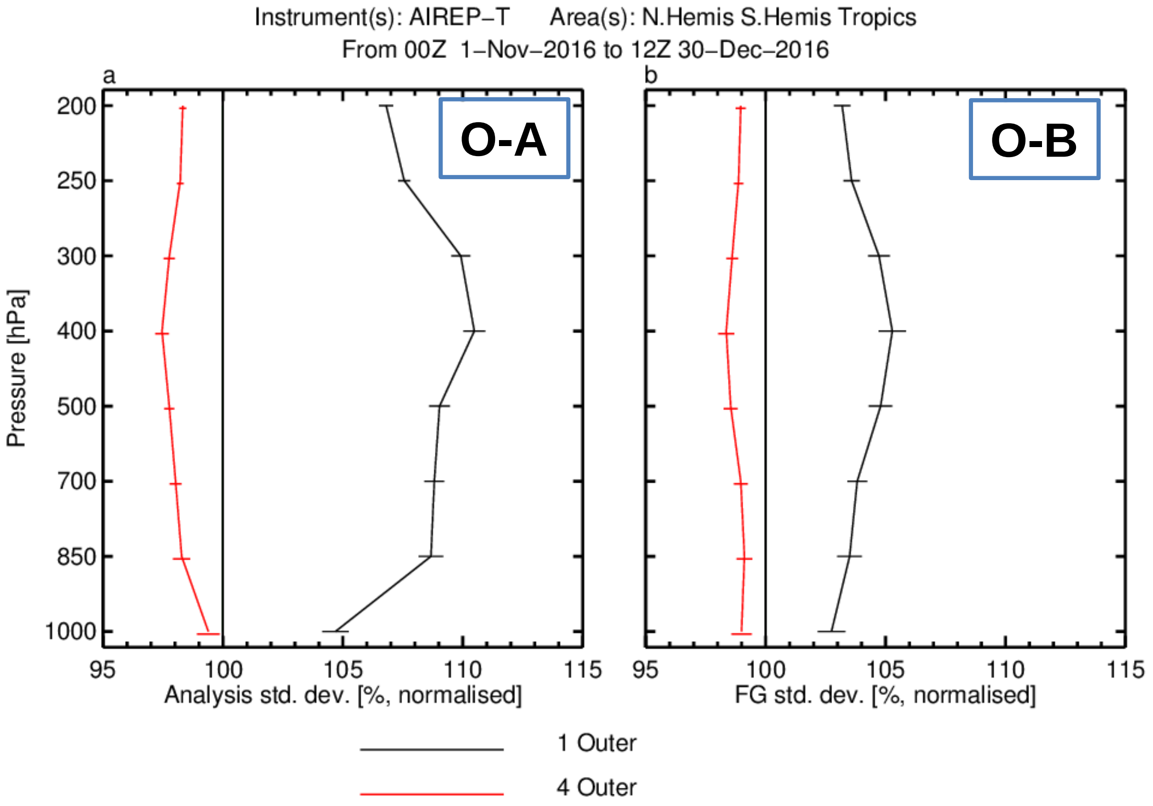
Nonlinear effects: the incremental approach

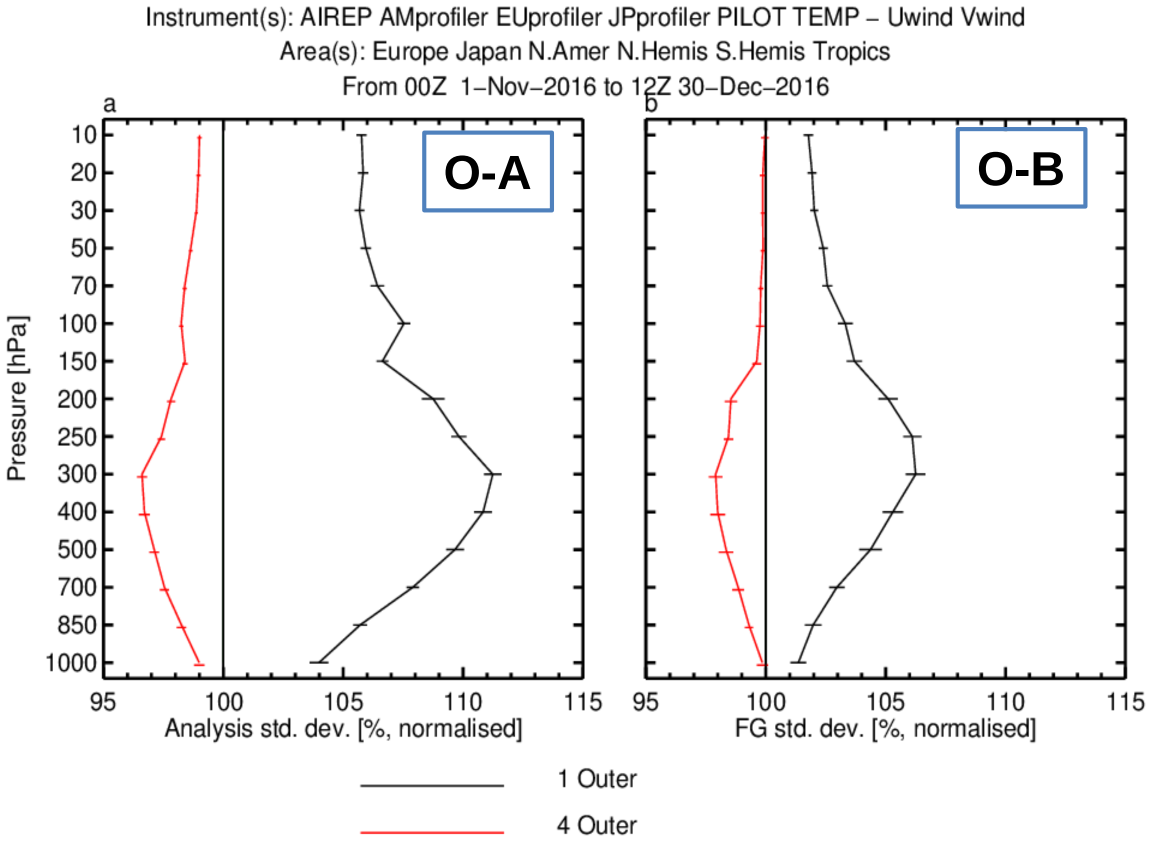
Taylor diagram for differences in obs departures from nonlinear and linearised trajectories



Nonlinear effects: the incremental approach

- How important is the capacity to run outer loops for global analysis and forecast skill?
- Relative difference of observation departures of 1 OL and 4 OL wrt 3OL control





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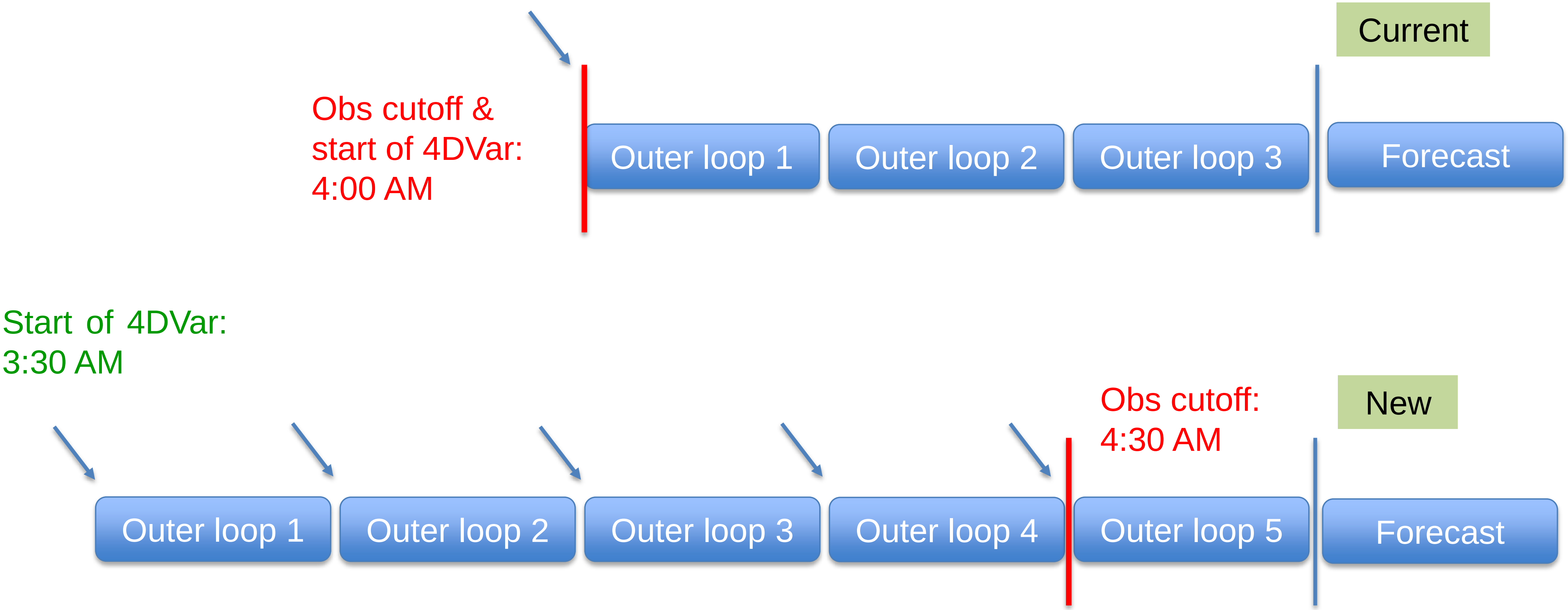
Nonlinear effects: implications for DA strategy

- Representativity and forward model errors are important and need to be modelled in R
- Nonlinearities (and non-Gaussianity) grow rapidly with fcst length and DA window length.
  - This requires more frequent analysis updates
  - If we want to keep the benefits of longer assimilation windows (better scores, easier Ocean coupling, etc.) some form of overlapping windows DA will be beneficial
- Being able to run multiple outer loops is crucial for analysis/fcst skill: how to fit them in a tight operational schedule?

- **Continuous DA approach**



P. Lean, E. Holm, M. Bonavita

Thanks for your attention!

Bonavita, M., M. Dahoui, P. Lopez, F. Prates, E. Hólm, G. De Chiara, A. Geer, L. Isaksen and B. Ingleby, 2017: On the Initialisation of Tropical Cyclones. ECMWF Tech. Memorandum. n. 810.

Bonavita, M., P. Lean and E. Hólm, 2018: Nonlinear effects in 4D-Var. Nonlin. Proc. Geo., in preparation