Preservation of physical properties with Ensemble-type Kalman Filter Algorithms

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Hans-Ertel-Zentrum
für Wetterforschung

Motivation

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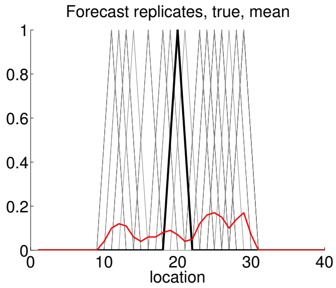
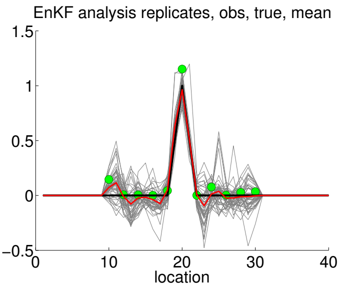
1. Explore which conservation properties are well recovered when using an ensemble Kalman filter
2. Include as constraints those that are not in data assimilation
3. Show implication on the prediction

Preserving physical properties





Preserving physical properties

Goal: Explore which conservation properties are well recovered when using LETKF

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- Non-linear dynamics with 2D nonlinear shallow water model
- Model settings:
  1. Mirror boundaries,
  2. constant $f = 0.0001$,
  3. 259×259 grid points with spacing 50km
  4. leapfrog scheme with time step 125s
  5. Asselin filter with 0.01.

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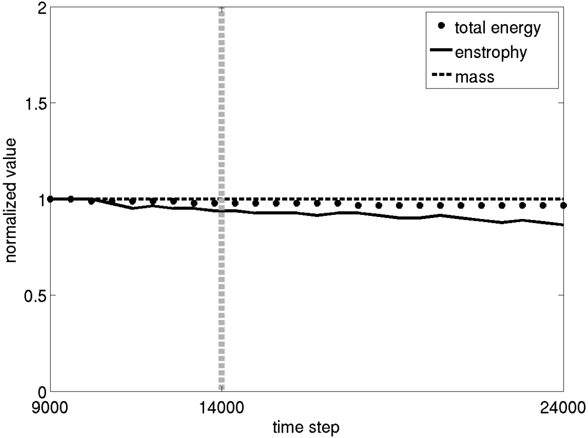
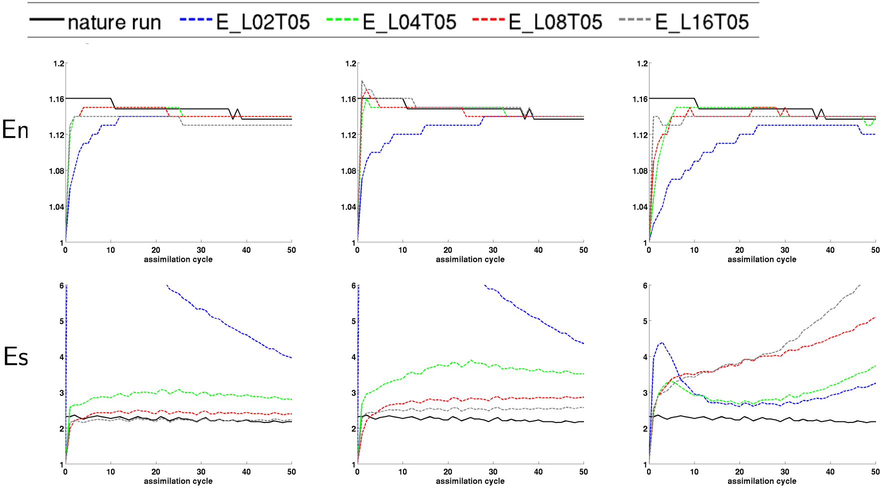
- Numerical discretization of the dynamics is such that mass, energy and momentum are conserved and enstrophy for non divergent flow.

Nonlinear shallow water model



Time evolution of mass, total energy and enstrophy, normalized with respective initial values, in a nature run.

Energy and Enstrophy

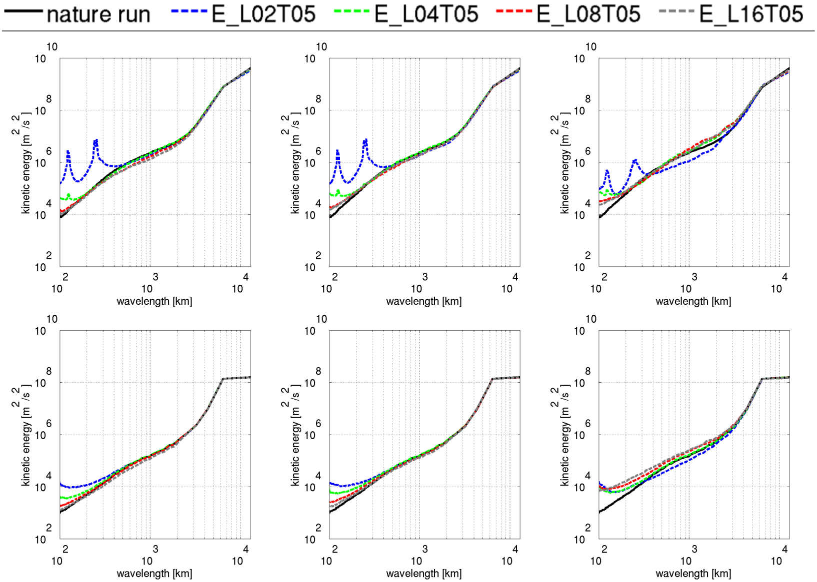


Obs u,v and h

Obs u and v

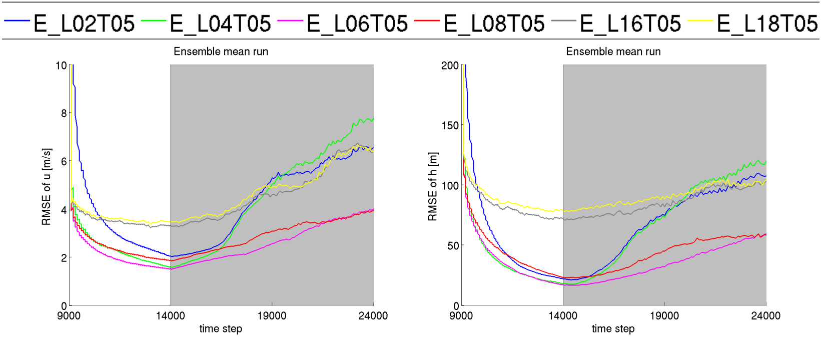
Obs h

Kinetic energy spectra



Averaged over the first (upper) and last five assimilation cycles (lower).

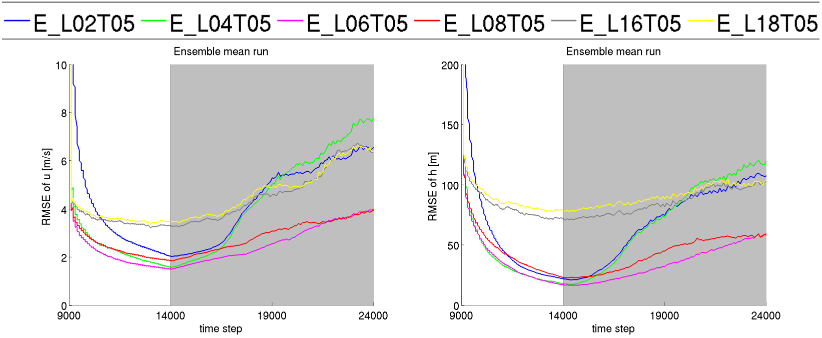
Prediction



RMSE for u

RMSE for h

Prediction



RMSE for u

RMSE for h

Y. Zeng and T. Janjic, 2016: Study of Conservation Laws with the Local Ensemble Transform Kalman Filter, Q. J. R. Meteorol. Soc.,142:699, 2359–2372.

— EnKF with constraints —

Janjic, T., D. McLaughlin, S. E. Cohn, M. Verlaan, 2014: Conservation of mass and preservation of positivity with ensemble-type Kalman filter algorithms, Mon. Wea. Rev., 142, No. 2, 755-773.

Zeng, Y., T. Janjić, Y. Ruckstuhl and M. Verlaan, 2017: Ensemble-type Kalman filter algorithm conserving mass, total energy and enstrophy, Q. J. R. Meteorol. Soc., 143:708, 2902–2914, doi:10.1002/qj.3142.

QPEns algorithm

Inverse of ensemble derived analysis error covariance can be used to minimize the cost function to obtain the analysis

$$\mathbf{w}_k^{a,i} = \mathbf{w}_k^{b,i} + \underset{\delta w^i}{\arg\min}\ \frac{1}{2}[\delta\mathbf{w}^{i\,T}(\mathbf{P}^b)^{-1}\delta\mathbf{w}^i + \mathbf{f}^{i\,T}\mathbf{R}^{-1}\mathbf{f}^i]$$

subject to

$$\delta\mathbf{w}^i \geq -\mathbf{w}_k^{b,i}.$$

where

$$\delta\mathbf{w}^i = \mathbf{w}_k^{a,i} - \mathbf{w}_k^{b,i}, \mathbf{f}^i = \mathbf{w}_k^{o,i} - \mathbf{H}_k\mathbf{w}_k^{b,i} - \mathbf{H}_k\delta\mathbf{w}^i - \bar{\mathbf{r}}_k^o.$$

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subject to

$$c_j(\delta \mathbf{w}_i) \leq 0,\ \ j \in \{1, 2, ..., m_1\}$$
$$g_k(\delta \mathbf{w}_i) = 0,\ \ k \in \{1, 2, ..., m_2\}$$

where

$$\delta\mathbf{w}^i = \mathbf{w}_k^{a,i} - \mathbf{w}_k^{b,i}, \mathbf{f}^i = \mathbf{w}_k^{o,i} - \mathbf{H}_k\mathbf{w}_k^{b,i} - \mathbf{H}_k\delta\mathbf{w}^i - \bar{\mathbf{r}}_k^o.$$

QPEns algorithm in ensemble space

$\rho = Rank(\mathbf{P}^b)$, which is no larger than $N-1$

$$\delta \mathbf{w}^i = \mathbf{L}\eta^i$$

$$\mathbf{P}^b = \mathbf{L}\mathbf{L}^T$$

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QPEns Algorithm in ensemble space

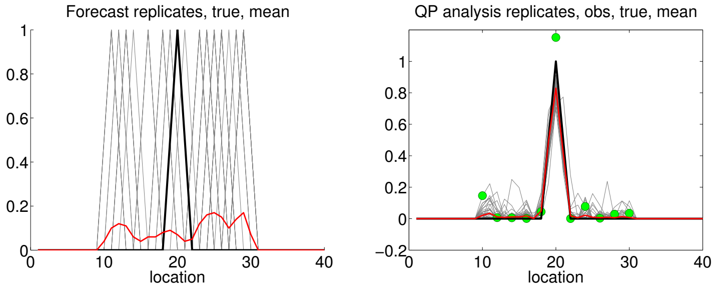
$$\eta^i = \arg\min_{\eta^i} \frac{1}{2}[{\eta^i}^T \eta^i + {\mathbf{f}^i}^T \mathbf{R}^{-1} \mathbf{f}^i]$$

subject to the following non-negativity constraint:

$$-\mathbf{L}\eta^i \leq \mathbf{w}_k^{f,i}.$$

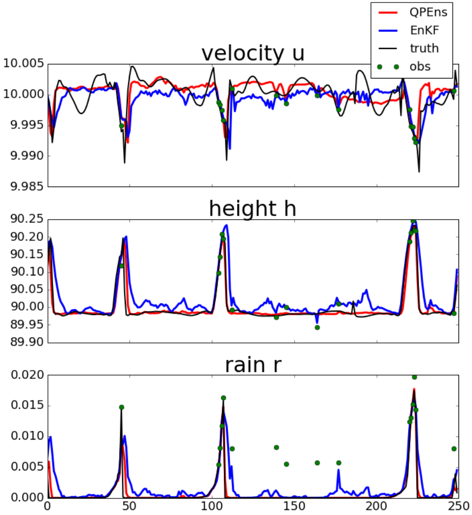
The algorithm reduces to EnKF if there are no constraints present.

Preserving physical properties



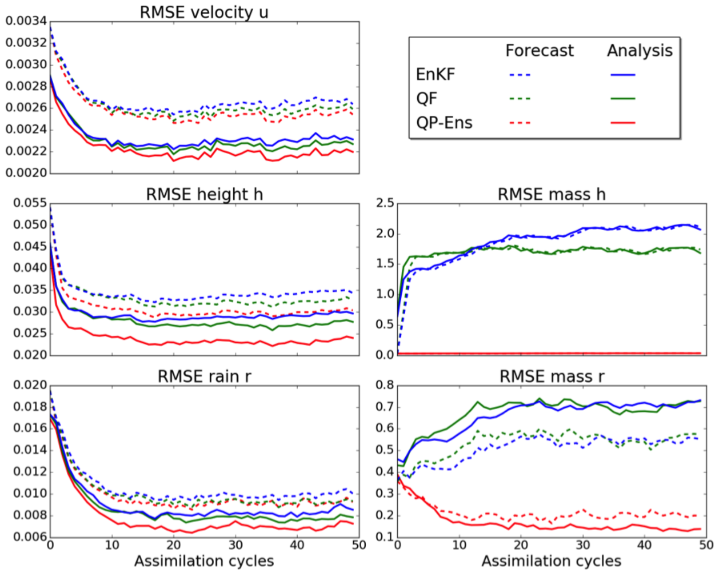
QPEns analysis in ensemble space with positivity constraint. Both mass conservation and positivity constraint improve analysis.

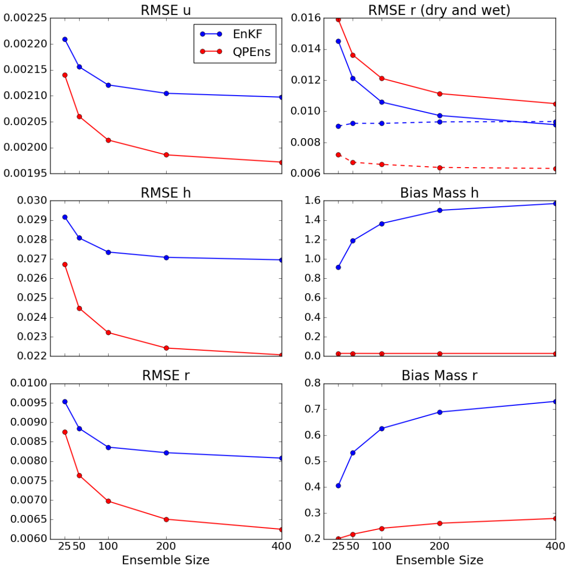
EnKF vs. QPEns



EnKF vs. QPEns analysis with positivity and mass constraint for modified shallow water model (Wuersch and Craig 2014).

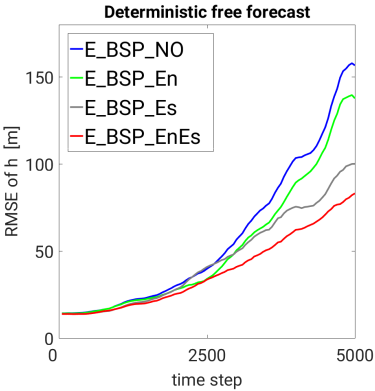
RMSEs


RMSE velocity u
RMSE height h
RMSE mass h
RMSE rain r
RMSE mass r
Forecast
Analysis
EnKF
QF
QP-Ens
Assimilation cycles
Assimilation cycles



Figures from Ruckstuhl and Janjic 2018: Parameter and state estimation with EnKF based algorithms for convective scale applications, QJRMS.

Prediction



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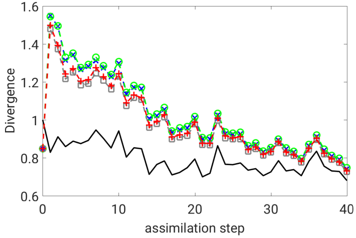
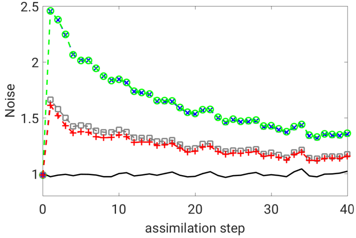
Diagnostics





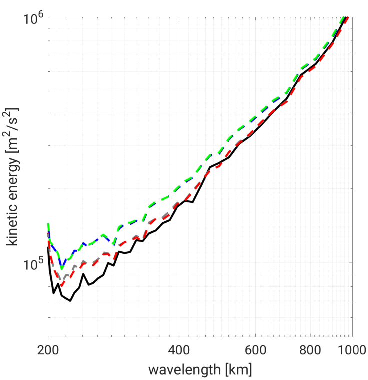
Divergence

Noise

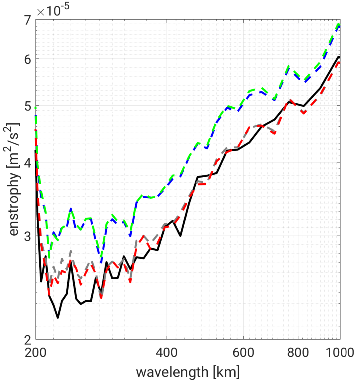
Variations of model diagnostics of divergence and noise within the data assimilation in experiments

E_BSP_NO E_BSP_En E_BSP_Es and E_BSP_EnEs.

Small scale spectra



Energy spectra



Enstrophy spectra

E_BSP_NO E_BSP_En E_BSP_Es E_BSP_EnEs.

Conclusion

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- Method is by construction multivariate
- Allows inclusion of other linear and nonlinear constraints
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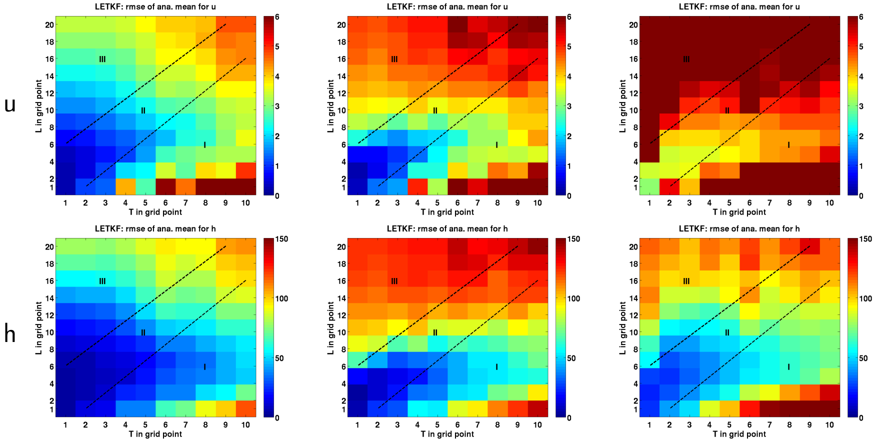
- Although total energy of the analysis ensemble mean converges towards the nature run value with time, enstrophy does not.
- Imposing the conservation of enstrophy within the data assimilation effectively avoids the spurious energy cascade of rotational part and this way succesfully suppresses the noise.
- Conserving mass and positivity reduces the noise in convective scale data assimilation applications.

RMSE



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