Nonlinear Bias Correction for Satellite Data Assimilation using A Taylor Series Polynomial Expansion of the Observation Departures

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Data Assimilation System

- Data assimilation experiments were performed using the Kilometer-scale Ensemble Data Assimilation (KENDA) system developed by the Deutscher Wetterdienst (DWD).

- COSMO model used as the NWP model with a model configuration similar to that used in the operational DWD system.

- Conventional observations (radiosondes, surface, wind profiler, and aircraft) were actively assimilated using a 1-h assimilation window.

- SEVIRI 6.2 μm infrared brightness temperatures were thinned by a factor of five and then passively monitored.

- Model-equivalent brightness temperatures computed using version 10.2 of the RTTOV radiative transfer model.

  - Assumed hexagonal ice crystals with the cloud particle diameters computed using the McFarquhar option.
Data Assimilation System

• Assimilation experiments performed on the COSMO-DE domain that covers Germany and surrounding areas with 2.8 km grid spacing

• 40 member ensemble initialized at 00 UTC on 16 May 2014 and then updated at hourly intervals during the next five days

• Passive monitoring statistics accumulated over a 4.5 day period starting at 12 UTC on 16 May 2014

• Clear and cloudy sky 6.2 μm brightness temperatures sensitive to clouds and water vapor in the middle and upper troposphere

  • Provides a spatially continuous 2-dimensional view of cloud and water vapor fields across entire model domain
Nonlinear Bias Correction Method

• Remove linear and nonlinear conditional biases from all-sky satellite observations using a Taylor series expansion of the OMB departures

• Linear bias corrections have been shown to work well for clear-sky satellite observations that have Gaussian error characteristics

• Nonlinear error dependencies are more likely to occur when cloudy observations are assimilated
  
  • Complex nonlinear cloud processes in the NWP model
  
  • Errors in the forward radiative transfer model used to compute the model-equivalent brightness temperatures

• Desirable to develop bias correction methods that can remove both the linear and nonlinear bias components from the observation departures
Nonlinear Bias Correction Method

• We define the observation departure vector as: \( \mathbf{d}y = y - H(x) \)

• If we assume that the bias can be described by a real function \( f(z) \) that is infinitely differentiable around a real number, \( c \), this equation can then be decomposed into an \( N \) order Taylor series expansion

• Single variable (e.g., predictor) case can be written as:

\[
\mathbf{d}y = \left( f(c) + \frac{f'(c)(z^{(i)} - c)}{1!} + \frac{f''(c)(z^{(i)} - c)^2}{2!} + \frac{f'''(c)(z^{(i)} - c)^3}{3!} + \ldots + \frac{f^{(n)}(c)(z^{(i)} - c)^n}{n!} \right)_{i=1,\ldots,m}
\]

0th Order (constant) 1\textsuperscript{st} Order (linear) 2\textsuperscript{nd} Order (quadratic) 3\textsuperscript{rd} Order (cubic) nth Order
Nonlinear Bias Correction Method

For a single variable, third order expansion with the bias correction coefficients defined as:

\[ b_n = \frac{f^{(n)}(a)}{n!} \]

\[ dy = \left( b_0 + b_1 (z^{(i)} - c) + b_2 (z^{(i)} - c)^2 + b_3 (z^{(i)} - c)^3 \right)_{i=1,\ldots,m} \]

This equation can be rewritten in matrix notation as: \[ dy = Ab \]

Because this kind of system typically does not have an analytic solution, we instead want to find the coefficients \( b \) that fit the equations by solving the quadratic minimization problem:

\[ S(b) = \sum_{i=1}^{m} |dy_i - \sum_{j=1}^{n} A_{ij} b_j|^2 = \|dy - Ab\|^2 \]
Nonlinear Bias Correction Method

After adding a Tikhonov regularization term to improve the conditioning:

$$\hat{S}(b) = \|d_y - Ab\|^2 + \alpha \|Ib\|^2$$

And then differentiating with respect to $b$ and setting the derivative to 0:

$$b = (\alpha I + A^T A)^{-1} A^T d_y$$

The inverse matrix is a symmetric, square matrix with dimensions $n \times n$

The small dimensions of this matrix make it easy to compute its inverse, thereby making it feasible to include higher order Taylor series terms, additional predictors, and a large observation departure dataset.
Observed SEVIRI 6.2 μm Brightness Temperatures

- Nice mixture of clear and cloudy scenes during the 5-day period
Observed 6.2 µm Brightness Temperature Predictor

- Results evaluated for original, 0\textsuperscript{th} (constant), 1\textsuperscript{st} (linear), 2\textsuperscript{nd} (quadratic), and 3\textsuperscript{rd} (cubic) order Taylor series expansions
- Purple line shows mean bias of the distribution
- Short black lines show conditional bias in each vertical column
- Used to assess how the bias varies as a function of the predictor value

- Each error distribution (except for the original) has zero overall bias; however, the conditional biases strongly vary as a function of the predictor value
Observed 6.2 μm Brightness Temperature Predictor

- Nonlinear conditional bias error pattern in the original distribution
- Constant and linear BC terms unable to remove all of the conditional bias
- Asymmetric arch shape in the conditional biases after 1\textsuperscript{st} order BC, which is removed after applying the 2\textsuperscript{nd} order BC
- Most of the remaining bias is removed after the 3\textsuperscript{rd} order BC is applied

- Though each departure distribution has zero overall bias, the conditional biases are much smaller when using higher order, nonlinear bias correction terms
Retrieved Cloud Top Height Predictor

- Nonlinear conditional bias error pattern in the original distribution
- Constant and linear BC terms unable to remove the conditional biases
- Arch pattern in the 1\textsuperscript{st} order conditional biases removed when using the 2\textsuperscript{nd} order quadratic term
- Some additional small reductions in the biases after using 3\textsuperscript{rd} order term

Cloud top height can serve as an effective bias predictor for infrared brightness temperatures when higher order Taylor series terms are used
100-700 hPa Vertically Integrated Water Predictor

- Less complex OMB departure pattern
- Upward slope removed after using 1st order bias correction term
- Subtle arch pattern is subsequently removed after using 2nd order term
- Predictor removed the conditional biases, but had smaller impact than predictors sensitive to cloud top height

• Vertical location of the cloud top is a more effective predictor of the bias than is the amount of water vapor and cloud condensate
Satellite Zenith Angle Predictor

- Widely used in operational BC methods
- Conditional biases close to zero across most of the original distribution
- Small upward bulge for zenith angles between 46 and 50 degrees; slight downward trend with increasing zenith angle
- 1st to 3rd order terms removed most of these conditional biases

Impact of this predictor on the overall statistics was negligible when compared to previous predictors sensitive to clouds and water vapor
Clear-Sky Only – Observed Brightness Temp Predictor

- Large systematic bias and linear trend in the original distribution
- Most of the conditional biases are removed using only the constant and 1st order terms
- Little impact when using higher order Taylor series terms

- Consistent with most existing bias correction methods that typically only use constant and linear corrections to remove the bias from clear-sky observations
Cloudy-Sky Only - Cloud Top Height Predictor

- Overall error pattern is very similar to the all-sky distributions
- Large conditional biases remain after constant and 1st order terms
- Biases are removed after the 2nd and 3rd order terms are applied
- Show that nonlinear biases are primarily associated with cloudy-sky observations

- The nonlinear bias correction method can effectively remove both linear and nonlinear biases from all-sky satellite observations
Thank you for your attention!