# Nonlinear Bias Correction for Satellite Data Assimilation using A Taylor Series Polynomial Expansion of the Observation Departures


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# *Data Assimilation System*

- Data assimilation experiments were performed using the Kilometer-scale Ensemble Data Assimilation (KENDA) system developed by the Deutscher Wetterdienst (DWD)

- COSMO model used as the NWP model with a model configuration similar to that used in the operational DWD system

- Conventional observations (radiosondes, surface, wind profiler, and aircraft) were actively assimilated using a 1-h assimilation window

- SEVIRI 6.2 μm infrared brightness temperatures were thinned by a factor of five and then passively monitored

- Model-equivalent brightness temperatures computed using version 10.2 of the RTTOV radiative transfer model

    - Assumed hexagonal ice crystals with the cloud particle diameters computed using the McFarquhar option

# *Data Assimilation System*

• Assimilation experiments performed on the COSMO-DE domain that covers Germany and surrounding areas with 2.8 km grid spacing

• 40 member ensemble initialized at 00 UTC on 16 May 2014 and then updated at hourly intervals during the next five days

• Passive monitoring statistics accumulated over a 4.5 day period starting at 12 UTC on 16 May 2014

• Clear and cloudy sky 6.2 μm brightness temperatures sensitive to clouds and water vapor in the middle and upper troposphere

- Provides a spatially continuous 2-dimensional view of cloud and water vapor fields across entire model domain

# ***Nonlinear Bias Correction Method***

- Remove linear and nonlinear conditional biases from all-sky satellite observations using a Taylor series expansion of the OMB departures

- Linear bias corrections have been shown to work well for clear-sky satellite observations that have Gaussian error characteristics

- Nonlinear error dependencies are more likely to occur when cloudy observations are assimilated

    - Complex nonlinear cloud processes in the NWP model

    - Errors in the forward radiative transfer model used to compute the model-equivalent brightness temperatures

- Desirable to develop bias correction methods that can remove both the linear and nonlinear bias components from the observation departures

# *Nonlinear Bias Correction Method*

- We define the observation departure vector as: $\mathbf{dy} = \mathbf{y} - H(\mathbf{x})$

- If we assume that the bias can be described by a real function f(z) that is infinitely differentiable around a real number, c, this equation can then be decomposed into an N order Taylor series expansion

- Single variable (e.g., predictor) case can be written as:

$$\mathbf{dy} = \left( f(c) + \frac{f'(c)(z^{(i)} - c)}{1!} + \frac{f''(c)(z^{(i)} - c)^2}{2!} + \frac{f'''(c)(z^{(i)} - c)^3}{3!} + \dots + \frac{f^{(n)}(c)(z^{(i)} - c)^n}{n!} \right)_{i=1,\dots,m}$$

0th Order (constant) | 1st Order (linear) | 2nd Order (quadratic) | 3rd Order (cubic) | nth Order

# Nonlinear Bias Correction Method

For a single variable, third order expansion with the bias correction coefficients defined as: $b_n = \frac{f^{(n)}(a)}{n!}$

$$\mathbf{dy} = \left(b_0 + b_1(z^{(i)} - c) + b_2(z^{(i)} - c)^2 + b_3(z^{(i)} - c)^3\right)_{i=1,\ldots,m}$$

This equation can be rewritten in matrix notation as: $\mathbf{dy} = \mathbf{Ab}$

Because this kind of system typically does not have an analytic solution, we instead want to find the coefficients **b** that fit the equations by solving the quadratic minimization problem:

$$S(b) = \sum_{i=1}^{m} \left|dy_i - \sum_{j=1}^{n} A_{ij} b_j\right|^2 = \|\mathbf{dy} - \mathbf{Ab}\|^2$$

# *Nonlinear Bias Correction Method*

After adding a Tikhonov regularization term to improve the conditioning:

$$\hat{S}(b) = \|\mathbf{dy} - \mathbf{Ab}\|^2 + \alpha \|I\mathbf{b}\|^2$$

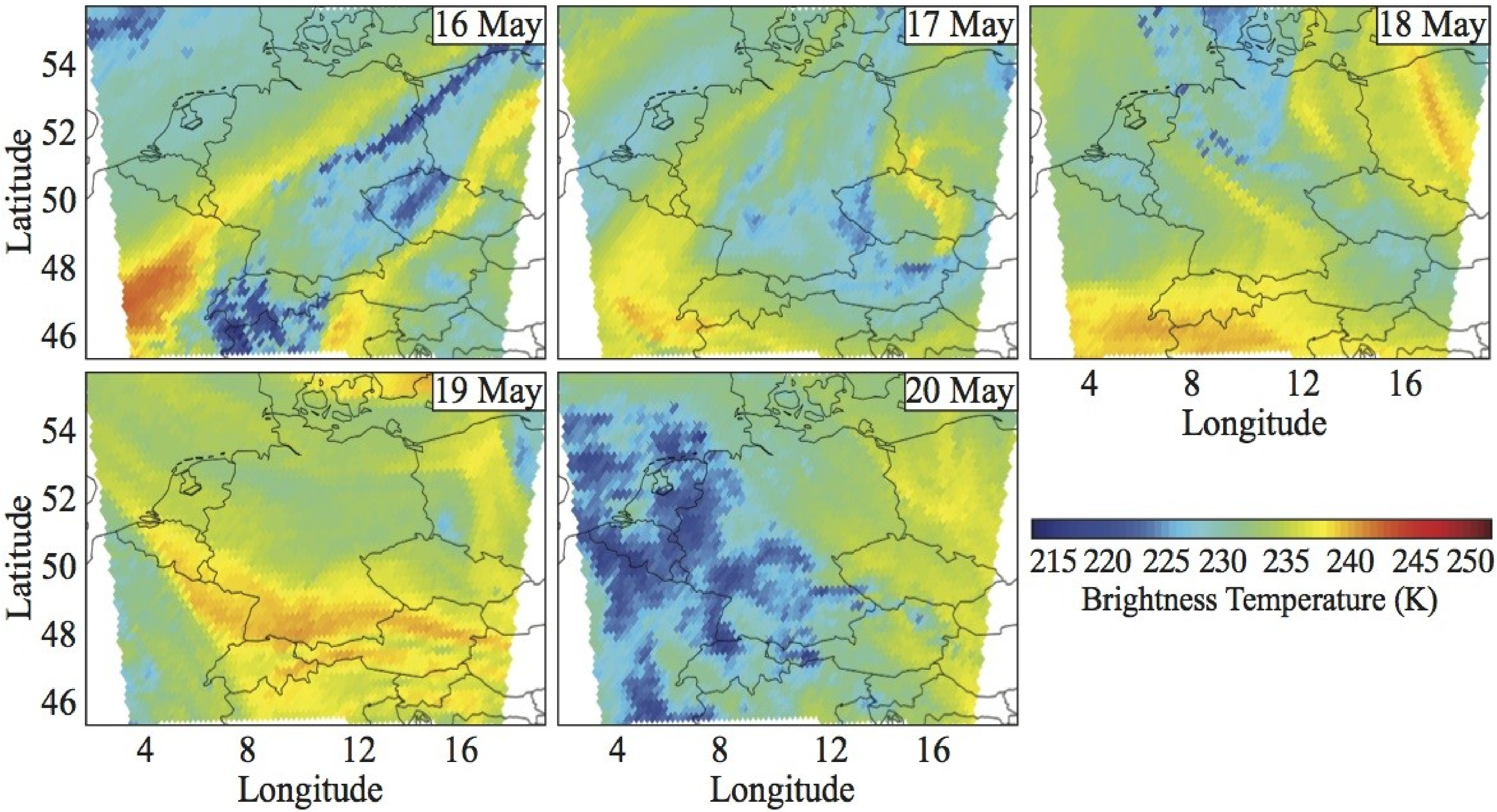
And then differentiating with respect to **b** and setting the derivative to 0:

$$\mathbf{b} = (\alpha I + \mathbf{A}^T\mathbf{A})^{-1}\mathbf{A}^T\mathbf{dy}$$

The inverse matrix is a symmetric, square matrix with dimensions n x n

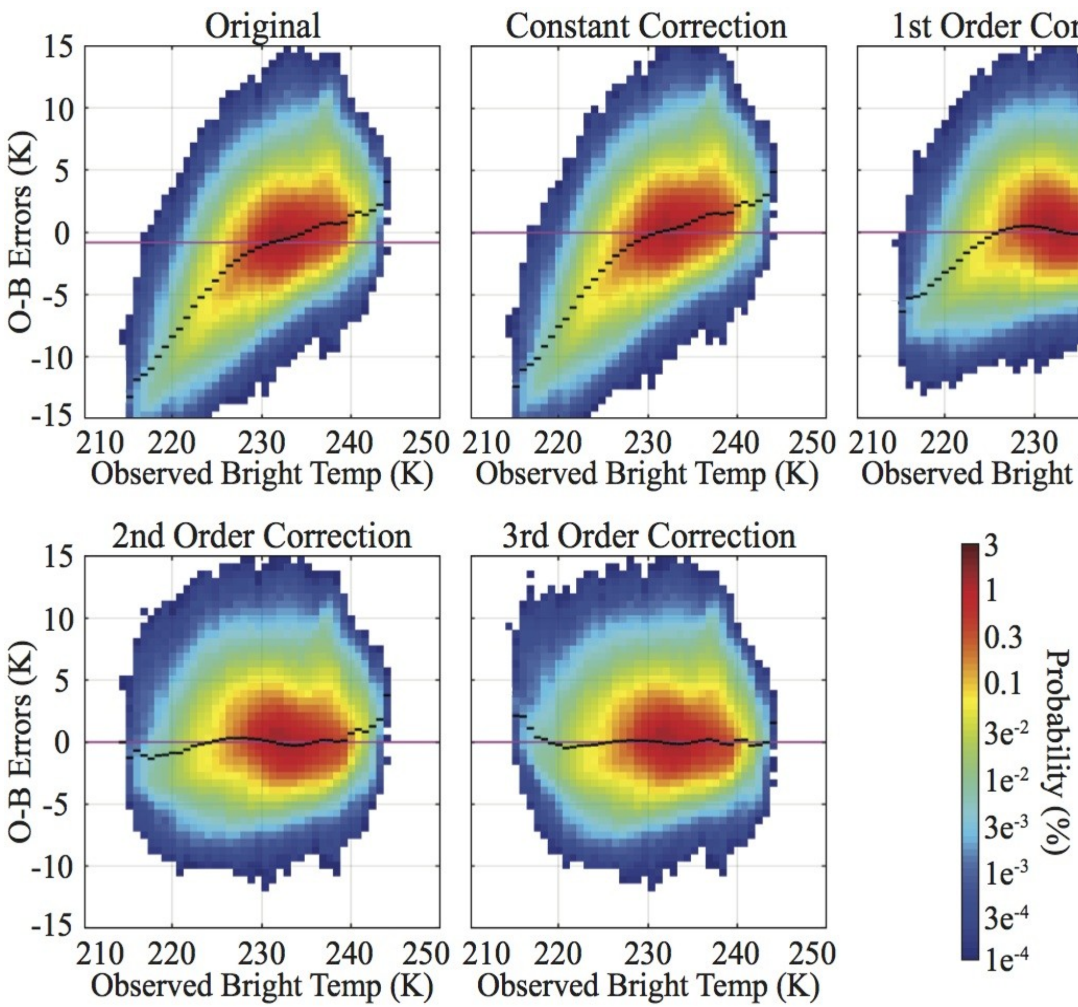
The small dimensions of this matrix make it easy to compute its inverse, thereby making it feasible to include higher order Taylor series terms, additional predictors, and a large observation departure dataset

# Observed SEVIRI 6.2 μm Brightness Temperatures




- Nice mixture of clear and cloudy scenes during the 5-day period

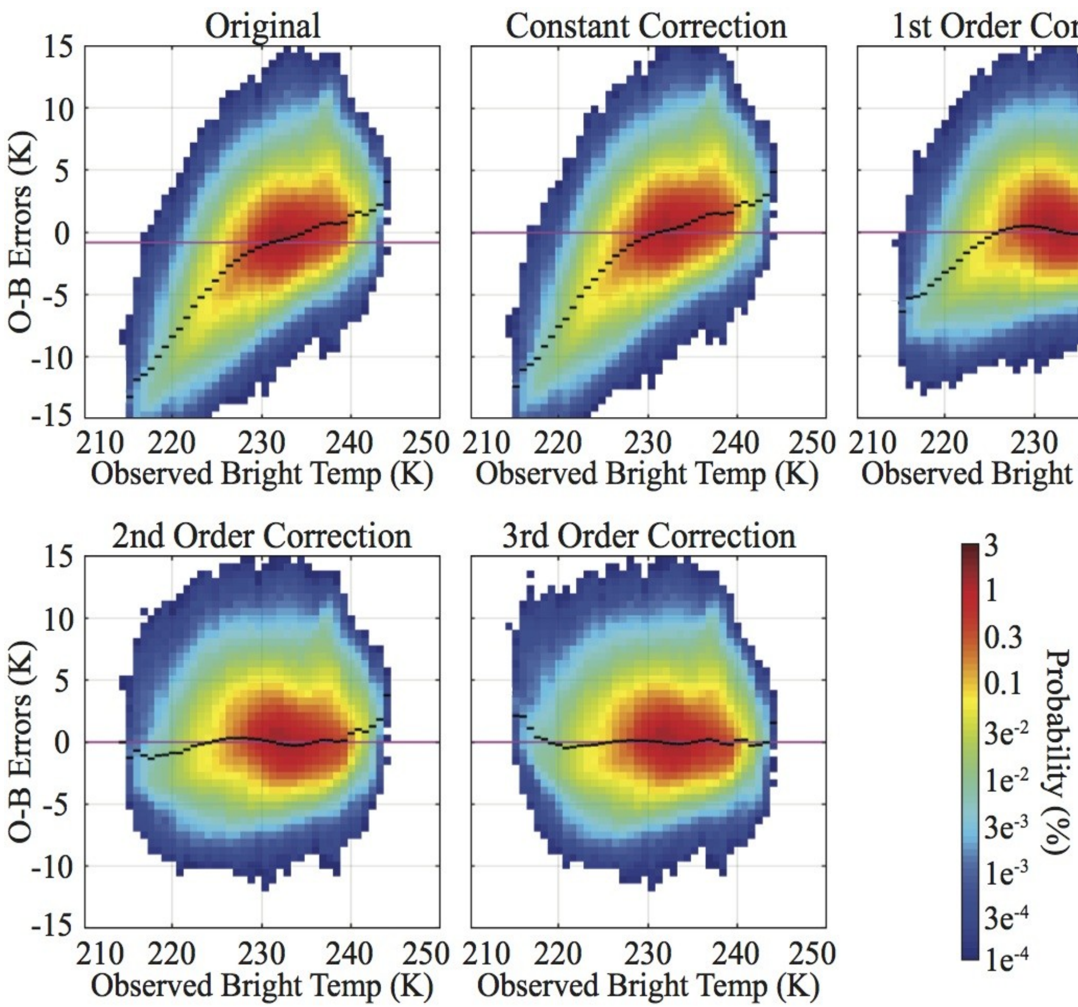
# Observed 6.2 μm Brightness Temperature Predictor




- Results evaluated for original, 0th (constant), 1st (linear), 2nd (quadratic), and 3rd (cubic) order Taylor series expansions
- Purple line shows mean bias of the distribution
- Short black lines show conditional bias in each vertical column
- Used to assess how the bias varies as a function of the predictor value

- Each error distribution (except for the original) has zero overall bias; however, the conditional biases strongly vary as a function of the predictor value

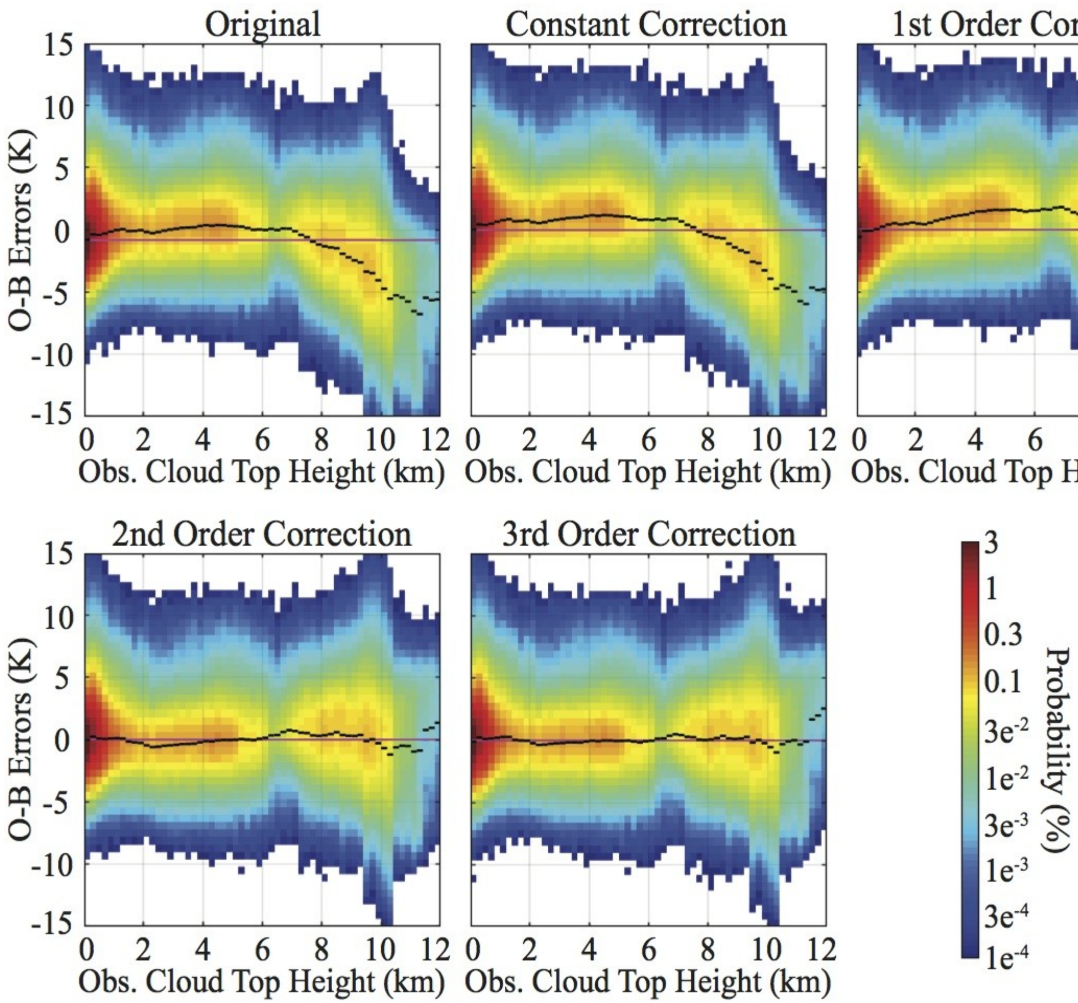
# *Observed 6.2 μm Brightness Temperature Predictor*




- Nonlinear conditional bias error pattern in the original distribution
- Constant and linear BC terms unable to remove all of the conditional bias
- Asymmetric arch shape in the conditional biases after 1st order BC, which is removed after applying the 2nd order BC
- Most of the remaining bias is removed after the 3rd order BC is applied

- Though each departure distribution has zero overall bias, the conditional biases are much smaller when using higher order, nonlinear bias correction terms

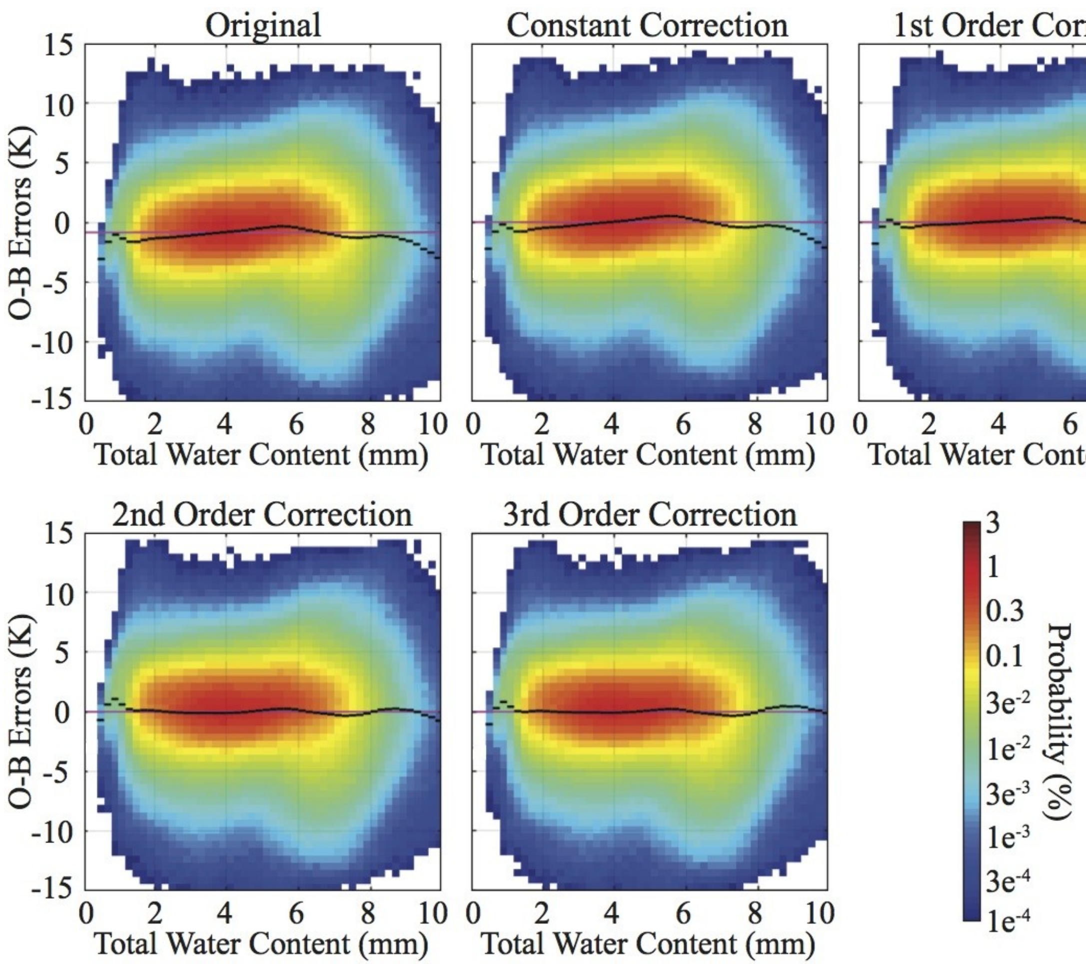
# Retrieved Cloud Top Height Predictor




- Nonlinear conditional bias error pattern in the original distribution
- Constant and linear BC terms unable to remove the conditional biases
- Arch pattern in the 1st order conditional biases removed when using the 2nd order quadratic term
- Some additional small reductions in the biases after using 3rd order term

- Cloud top height can serve as an effective bias predictor for infrared brightness temperatures when higher order Taylor series terms are used

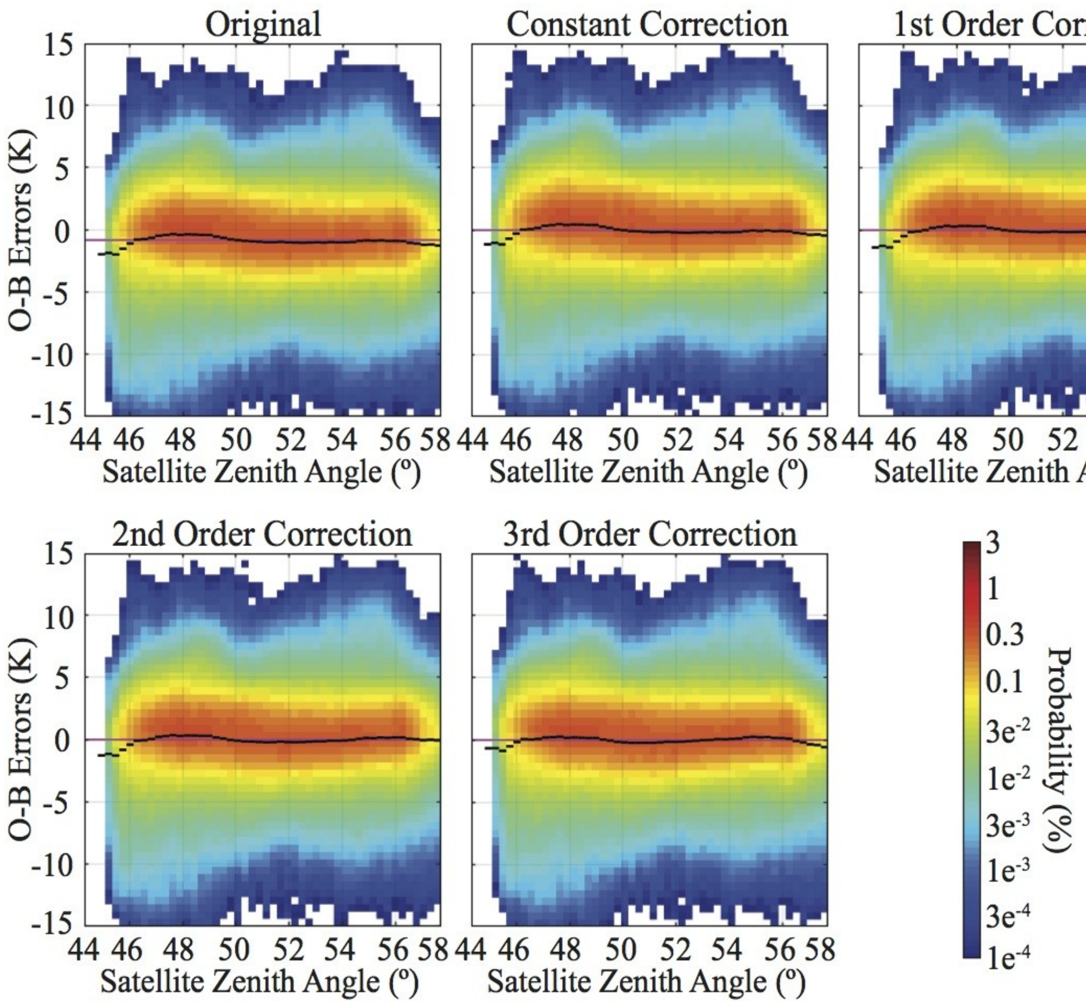
# 100-700 hPa Vertically Integrated Water Predictor




- Less complex OMB departure pattern
- Upward slope removed after using 1st order bias correction term
- Subtle arch pattern is subsequently removed after using 2nd order term
- Predictor removed the conditional biases, but had smaller impact than predictors sensitive to cloud top height

- Vertical location of the cloud top is a more effective predictor of the bias than is the amount of water vapor and cloud condensate

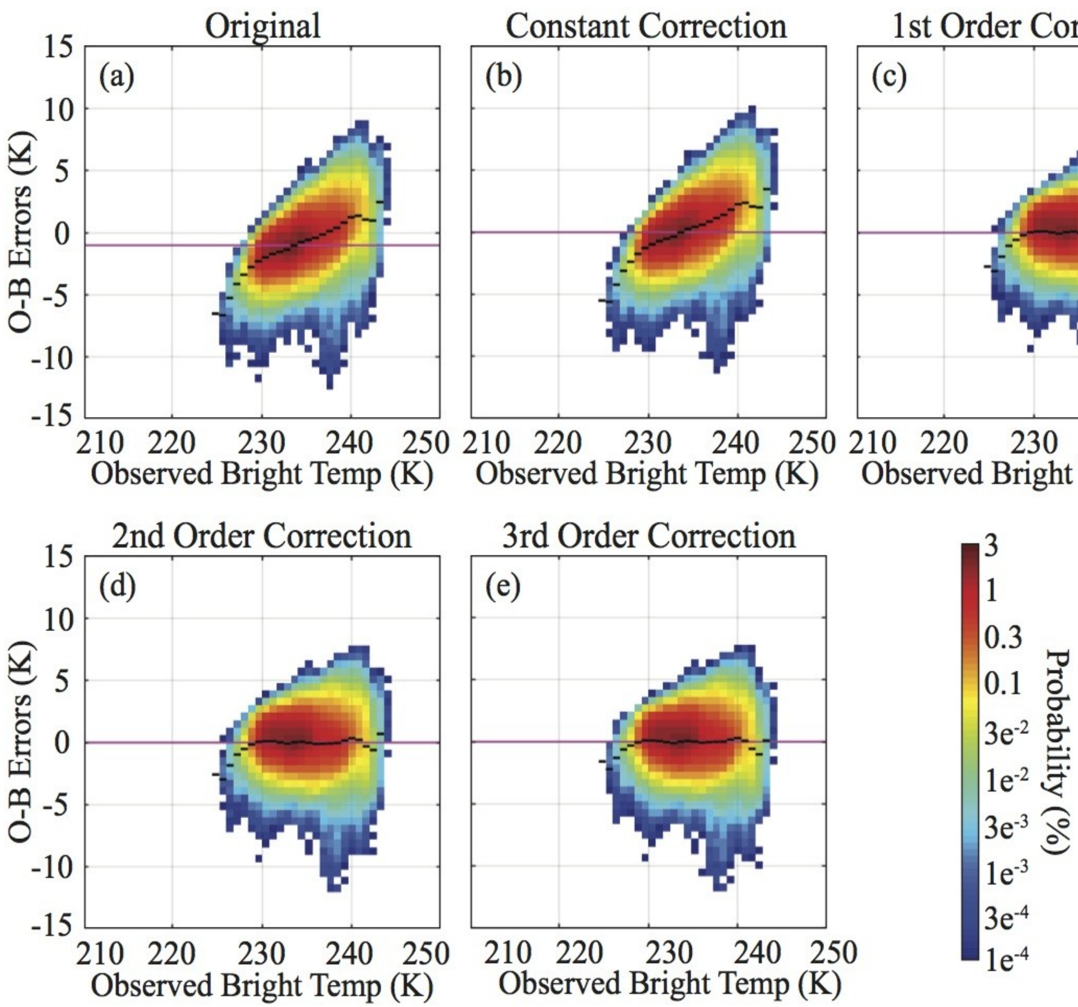
# Satellite Zenith Angle Predictor




- Widely used in operational BC methods
- Conditional biases close to zero across most of the original distribution
- Small upward bulge for zenith angles between 46 and 50 degrees; slight downward trend with increasing zenith angle
- 1st to 3rd order terms removed most of these conditional biases

- Impact of this predictor on the overall statistics was negligible when compared to previous predictors sensitive to clouds and water vapor

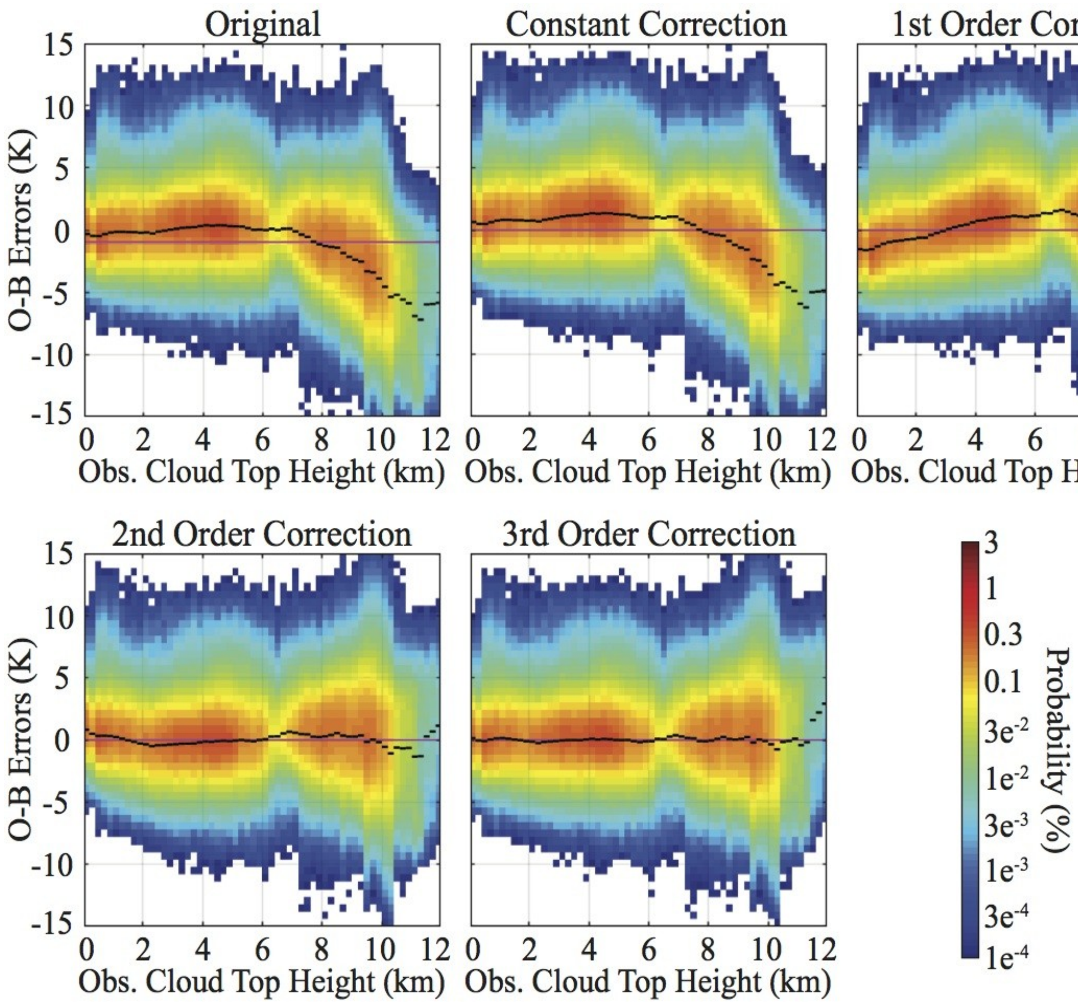
# Clear-Sky Only – Observed Brightness Temp Predictor




- Large systematic bias and linear trend in the original distribution
- Most of the conditional biases are removed using only the constant and 1st order terms
- Little impact when using higher order Taylor series terms

- Consistent with most existing bias correction methods that typically only use constant and linear corrections to remove the bias from clear-sky observations

# Cloudy-Sky Only – Cloud Top Height Predictor




- Overall error pattern is very similar to the all-sky distributions
- Large conditional biases remain after constant and 1st order terms
- Biases are removed after the 2nd and 3rd order terms are applied
- Show that nonlinear biases are primarily associated with cloudy-sky observations

- The nonlinear bias correction method can effectively remove both linear and nonlinear biases from all-sky satellite observations

# ***Thank you for your attention!***

Otkin, J. A., R. Potthast, and A. Lawless, 2018: Nonlinear bias correction for satellite data assimilation using Taylor series polynomials. *Mon. Wea. Rev.*, **146**, 263-285.